UNIQUE FACTORIZATION IN ALGEBRAIC FUNCTION FIELDS[1]

BY

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Introduction

Let K be a field of algebraic functions of one variable over a field k. Only those subdomains R of K which properly contain k are considered.

A preliminary result on quotient rings with respect to a multiplicative system is applied to the particular case that K is of genus zero to determine the conditions under which a given integrally closed subdomain of K is a quotient ring of a selected ring of a particularly simple type. This, in connection with a criterion that R be a unique factorization domain, yields a description of all subdomains of K which are unique factorization domains. The restriction on the genus of K removed, it is shown that under suitable conditions if R is a unique factorization domain or possesses certain kinds of prime elements, then K is of genus zero.

The author wishes to express his appreciation to Professor Abraham Seidenberg for his guidance in the preparation of this work and to the referee for several helpful suggestions.

1. Preliminaries

If K is a field of algebraic functions of one variable over a field k, it shall always be assumed that k is algebraically closed in K.

The definitions of place, valuation, zero, pole, divisor, and related terms are those of Chevalley [1]. Note, in particular, that a place is the ideal of non-units of a valuation ring.

A *Krull domain* is an integral domain R with unity such that there exists a family V of valuations of the quotient field F of R which are discrete and of rank 1, and such that R is the intersection of all valuation rings of valuations of V, and every nonzero element of F has zero value in all but a finite number of valuations of V. V is called a *definition family* of R. A valuation v in V is *essential* if there is an element x in F such that $v(x)$ is negative, but x has nonnegative value in every other valuation of V. The basic facts about Krull domains are to be found in Samuel [2], where they are called "normal" rings.

A *Dedekind domain* is a Krull domain in which every nontrivial prime ideal is minimal. Occasionally, for expository purposes, a domain, instead of being called simply a Dedekind domain, will be referred to as both a Krull and Dedekind domain.

Received March 21, 1963.

[1] This paper contains part of a doctoral dissertation written under the direction of Professor Abraham Seidenberg at the University of California, Berkeley.