ON THE CONTINUITY OF LATTICE AUTOMORPHISMS ON CONTINUOUS FUNCTION LATTICES

BY

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1. Introduction

Let E be a compact Hausdorff space, $C(E)$ the lattice of all real-valued continuous functions on E, and let $T : f \to f^T$ be a lattice automorphism of $C(E)$.

I. Kaplansky has proved in [2] the following two results.

(I) If T is homeomorphic in the topology of uniform convergence, then T can be characterized in the following form:

$$f^T(x^t) = \Phi(f(x), x) \qquad (x \in E, f \in C(E))$$

where $x \to x^t$ is a homeomorphism of E, and $\Phi(\xi, x)$ ($\xi \in \mathbf{R}$, $x \in E$) is a continuous function on $\mathbf{R} \times E$, and for any fixed $x \in E$, $\Phi(\;, x)$ is a lattice automorphism of $\mathbf{R}$.

(II) If E satisfies a first axiom of countability, then all lattice automorphisms of $C(E)$ are homeomorphic in the topology of uniform convergence. However, generally speaking, lattice automorphisms are not necessarily continuous.

It may be natural to consider the following problem: What is the characteristic topological property of E in order that all lattice automorphisms of $C(E)$ be continuous in the topology of uniform convergence?

In view of this problem the following three classes of compact Hausdorff spaces are considered.

(1) E has property (K): All lattice automorphisms of $C(E)$ are continuous.

(2) E has property (K_0): All compact subspaces of E have property (K).

(3) E has property (K_1): A lattice automorphism T of $C(E)$ is continuous if and only if T^{-1} is continuous.

The above three classes obviously satisfy the relations

$$(K_0) \subset (K) \subset (K_1).$$

Our purpose in this paper is to give a complete topological characterization of properties (K_0) and (K_1).

THEOREM 1. *E has property (K_1) if and only if $E \neq \beta U$ for any dense open F_σ-subset $U \subset E$, $U \neq E$, where βU is a Stone-Čech compactification*[1] *of U.*

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[1] For the definition and the fundamental properties of Stone-Čech compactification the reader is referred to [3, Chapter 6].