GLOBAL SECTIONS OF TRANSFORMATION GROUPS

BY

ROBERT ELLIS[1]

Let (X, R) be a transformation group with phase space X and phase group R, the additive group of real numbers. Suppose further that (X, R) is minimal. Then what can be said about X? Various answers have been given to this question, see for example [4], [5], [6], [11], [12]. In [12] Schwartzman shows that if in addition X is compact, locally pathwise connected, and if (X, R) admits a global section, then X is the base of a covering space with discrete fibers. This allows him to say something about the homotopy groups of X. In particular he shows that $\pi_1(X) \neq 0$. Recently Chu and Geraghty [5] showed that if X is compact, locally pathwise connected, and if (X, R) is minimal but not totally minimal, then $\pi_1(X) \neq 0$.

The first part of this paper is devoted to generalizing the notion of global section. The above results are considered in a more general setting, and the relation between them is studied. They are generalized to the case where R is replaced by any topological group whose underlying space is R^n.

The second part of the paper is concerned with the following problem. Suppose X is a manifold which is minimal under R; need X be orientable? This question is answered in the negative by exhibiting an action of R on the cartesian product X of the torus with the Klein bottle such that (X, R) is minimal. The flow is constructed by first producing a homeomorphism f of $S^1 \times K$ (the circle cross the Klein bottle) such that $S^1 \times K$ is minimal under the resulting discrete flow, and then R is allowed to act on $(S^1 \times K \times I)/f$ in the standard way; here I is the unit interval and $(S^1 \times K \times I)/f$ is obtained from $S^1 \times K \times I$ by identifying $(z, 0)$ with $(f(z), 1)$ $(z \in S^1 \times K)$. Since f turns out to be isotopic to the identity, the resulting space is homeomorphic to the cartesian product of the torus with the Klein bottle. This flow may be lifted to a flow on the four-torus, T^4. From a result of Auslander and Hahn [1] this flow does not come from a one-parameter subgroup of T^4.

For the remainder of this paper R will denote the additive group of real numbers, and Z the additive group of integers. Let (X, Z) be a transformation group with phase group Z. Then the action of Z on X is completely determined by the homeomorphism f of X onto X, where $f(x) = x1$ $(x \in X)$. For this reason the transformation group (X, Z) will often be denoted (X, f).

For a general discussion of the notions used see [9].

DEFINITION 1. A *left* [*right*] *transformation group* is a pair (G, X) $[(X, G)]$ where X is a topological space and G is a topological group together with a continuous map $(g, x) \to gx$ $[(x, g) \to xg]$ $(x \in X, g \in G)$ from $G \times X \to X$

Received February 18, 1963.

[1] The author is a National Science Senior Postdoctoral Fellow.