# CORRECTION TO MY PAPER "FIXED-POINT THEOREMS FOR COMPACT CONVEX SETS"[1]

BY

MAHLON MARSH DAY[2]

Mr. Neil Rickert has pointed out to me that the remark on page 589 claiming validity for the converse of Theorem 3 stumbles on the fact that the adjoint mapping of the left regular representation of $S$ over $C(S)$ need not be ($w^*$-) continuous unless $S$ is discrete or compact. (For example, over the real additive group $G$, a $\mu$ in $C(G)^*$ and an $x$ in $C(G)$ can easily be constructed such that $\mu(x) = [l_0{}^*\mu](x) = 1$ but $[l_g{}^*\mu](x) = 0$ for all $g \neq 0$). Because the proof of Theorem 1 anchors on one element of $K$, this need not be catastrophic. The converse as claimed is false, but a related condition is equivalent to left-amenability of $C(S)$. Recall that $A(K)$ is the semigroup of affine, continuous mappings of the compact convex set $K$ into itself.

DEFINITION. A homomorphism $\tau$ of a topological semigroup $S$ into $A(K)$ is called *slightly continuous* if and only if there is at least one $y$ in $K$ such that $[\tau s](y)$ is a continuous function from $S$ into $K$.

The remark at the bottom of (c), p. 589, should then be replaced by

THEOREM 4. *If $S$ is a topological semigroup, there is a left-invariant mean on $C(S)$ if and only if for each compact convex set $K$ in each locally convex space $X$ and for each slightly continuous homomorphism of $X$ into $A(K)$ there is in $K$ a common fixed point $p$ of all the transformations in the semigroup $\Sigma = \tau(S)$.*

*To sketch the proof*: If $\gamma$ is a left-invariant mean on $C(S)$, let $\delta$ be any extension of $\gamma$ which is a mean on $m(S)$. (For existence of $\delta$ see reference [4, Theorem 1, p. 20].) Define $V$ from $m(\Sigma)$ to $m(S)$ by: for all $x$, $[Vx](s) = x(\tau s)$ for all $s$. If $\mu = V^*\delta$, then $\mu$ is a mean on $m(\Sigma)$. If $y$ is chosen in $K$ to fit the condition imposed by slight continuity of $\tau$, the left-invariance of $\gamma$ on $C(S)$ implies that $h(j\mu) = j\mu$ for every $h$ in $\Sigma$; that is, that $j\mu = jV^*\delta$ is the desired fixed point in $K$.

The proof for the converse requires only the following lemma, which can be proved by observing that every evaluation functional $\eta_{s_0}$, defined by $\eta_{s_0}(x) = x(s_0)$ for all $x$ in $C(S)$, is a mean on $C(S)$ for which $l_s{}^*\eta_{s_0}$ is a continuous function from $S$ into $C(S)^*$ with its $w^*$-topology.

*Lemma. The adjoint of the left regular representation over $C(S)$ is a slightly continuous representation of $S$ in $A(M)$, when $M$ is the ($w^*$-compact, convex) set of means on $C(S)$ with the $w^*$-topology.*

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