APPLICATIONS OF A COMPACTIFICATION FOR BOUNDED OPERATOR SEMIGROUPS

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Introduction and Summary

In this paper[1] a compactification for bounded semigroups of linear operators in a Banach space is studied and some applications to abstract ergodic theory and invariant means are given. In Sec. 1 the compactification in question is described and in Sec. 2 its ideal theory is developed. Sec. 3 contains a discussion of ergodic elements for arbitrary bounded, not necessarily "ergodic," operator semigroups and is very close in spirit to Eberlein [6]. The connection between the compactification and the convolution semigroup of means introduced by Day in [4] is established (in (4.3)) and the following theorem is proved: the space $m(\Sigma)$ of all bounded real functions on an abstract semigroup Σ with unit contains a largest right amenable right introverted subspace Z which, moreover, lies in every maximal right amenable subspace of $m(\Sigma)$.

The following notations will be used throughout: If B_1, B_2 are Banach spaces then B_1^* is the conjugate space of B_1 and $L(B_1, B_2)$ is the Banach space of all bounded linear operators of B_1 into B_2; if $S \subset L(B_1, B_2)$ and $x \in B_1$ then $O_S(x)$ is the orbit of x under S and defined by $O_S(x) = \{Ax : A \in S\}$. The closure of a set S is denoted by S^-, and composition is indicated by juxtaposition or brackets.

1. Compactification of a bounded operator semigroup

We need the following two devices.

I. Suppose X is a linear topological space and S is a semigroup (under composition) of continuous linear operators in X. Let S^- be the closure of S in the product space X^X. We have

(i) S^- is a semigroup (under composition) of linear operators in X, and

(ii) for fixed $A \in S$ and $B \in S^-$ the maps $F \to AF$ and $F \to FB$ $(F \in S^-)$ are continuous in the product topology of X^X.

II. Suppose B is a Banach space. B can be regarded as a subspace of B^{**} and hence $L(B, B)$ can be regarded as a subspace of $L(B, B^{**})$. Let η be the mapping which takes each $U \in L(B, B^{**})$ into the function $F = \eta(U) \in L(B^*, B^*)$ defined by

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