THE CONSTRUCTION OF A CLASS OF DIFFUSIONS

BY
DONALD A. DAWSON

1. Introduction

E. B. Dynkin [4] has shown that the generator of a diffusion on a locally compact, separable space Q has a canonical representation in terms of the mean hitting times and hitting probabilities. Let $x(t)$ be a strict Markov process with generator $\mathfrak{G}$ whose domain is $D(\mathfrak{G})$. Let $f \in D(\mathfrak{G})$, $\xi \in Q$, U be a neighborhood of ξ with compact closure and nonnull boundary and τ^U be defined as inf $(t : x(t) \notin U)$. Then

$$(\mathfrak{G}f)(\xi) = \lim_{U \downarrow \xi} \frac{E_\xi(f(x(\tau^U))) - f(\xi)}{E_\xi(\tau^U)} .$$

It is easy to show that $\mathfrak{G}$ satisfies a maximum property and is a local operator on $C(Q)$. W. Feller [6] has posed the converse question, namely, does every local operator on $C(Q)$ which satisfies the maximum property generate a diffusion. As a partial solution of this problem it will be shown that every such operator arising from a set of mean hitting times and hitting probabilities having certain smoothness properties does indeed generate a diffusion. The method employed is the construction of a sequence of approximating random walks which will be shown to converge to a limit process which is a diffusion. This is an extension of the construction of F. B. Knight [10], [11] for the one-dimensional case.

This paper is based on the author's Ph.D. thesis written under the supervision of Professor Henry P. McKean Jr.

2. Some definitions and the main result

Let Q be a locally compact, separable Hausdorff space with metric $\rho(\cdot, \cdot)$. Let $\mathbf{C}$ be the class of all compact subsets of the state space Q and $\mathbf{S}$ be the σ-field generated by $\mathbf{C}$. The sets of $\mathbf{S}$ are called the *Borel sets* of Q [7].

Let Δ be a collection of open sets with nonnull boundaries of the space Q such that

i. the closure of any set of Δ is a compact subset of Q,
ii. Δ is a base for the topology of Q, and
iii. if D_1, $D_2 \in \Delta$, then $D_1 \cup D_2$, $D_1 - \bar{D}_2$, and $D_1 \cap D_2 \in \Delta$ if they are nonempty.

For $D \in \Delta$, let $\mathbf{B}(\partial D)$ be the class of Borel subsets of ∂D, the boundary of D.

Received July 22, 1963.