FINITE MÖBIUS-PLANES ADMITTING A ZASSENHAUS GROUP AS GROUP OF AUTOMORPHISMS[1]

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We call an incidence structure $\mathfrak{M}$ consisting of points and circles and an incidence relation between points and circles a Möbius-plane (= inversive plane), if the following axioms are satisfied (see e.g. Benz [1]):

(1) *If P, Q, R are three different points of $\mathfrak{M}$, then there exists one and only one circle k in $\mathfrak{M}$ such that $P, Q, R \in k$.*

(2) *If k is a circle and P a point on k and if Q is a point not on k, then there exists one and only one circle l with $P, Q \in l$ and $k \cap l = \{P\}$.*

(3) *There are four points which do not all lie on the same circle, and every circle carries at least one point.*

σ is called an *automorphism* of $\mathfrak{M}$, if σ is a permutation of the points of $\mathfrak{M}$ which maps concyclic points on concyclic points. The full automorphism group of $\mathfrak{M}$ is called the *Möbius-group* of $\mathfrak{M}$.

If P is a point of $\mathfrak{M}$, then we derive an incidence structure $\mathfrak{A}(\mathfrak{M}, P)$ from $\mathfrak{M}$ and P in the following way:

(a) *The points of $\mathfrak{A}(\mathfrak{M}, P)$ are the points of $\mathfrak{M}$ which are different from P.*

(b) *The lines of $\mathfrak{A}(\mathfrak{M}, P)$ are the circles through P.*

(c) *A point Q and a line l of $\mathfrak{A}(\mathfrak{M}, P)$ are incident if and only if the corresponding point Q and the corresponding circle l are incident in $\mathfrak{M}$.*

It is a well known fact that $\mathfrak{A}(\mathfrak{M}, P)$ is an affine plane (Benz [1, Satz 1]).

If $\mathfrak{M}$ is a finite Möbius-plane, then it follows from the fact that $\mathfrak{A}(\mathfrak{M}, P)$ is an affine plane that the number of points of $\mathfrak{M}$ is $q^2 + 1$ and the number of points which lie on a circle is $q + 1$. It is easily seen that the number of circles is $q(q^2 + 1)$. We call q the *order* of $\mathfrak{M}$.

Let $\mathfrak{B}$ be a set of circles and P a point. We call $\mathfrak{B}$ a *tangent bundle* through P, if the following hold:

(i) $\mathfrak{B} \neq \emptyset$.

(ii) $k, l \in \mathfrak{B}$ and $k \neq l$ imply $k \cap l = \{P\}$.

(iii) $k \in \mathfrak{B}$ and $k \cap l = \{P\}$ imply $l \in \mathfrak{B}$.

Let Σ be a permutation group on the set $\mathfrak{P}$; then we call Σ *Zassenhaus transitive* on $\mathfrak{P}$, if Σ is doubly transitive on $\mathfrak{P}$ and if only the identity fixes three different elements of $\mathfrak{P}$.

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