REMARKS ON NONLINEAR FUNCTIONAL EQUATIONS, II

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Introduction

In a series of recent papers, the writer [1]–[13] and G. J. Minty [15]–[17] have studied nonlinear functional equations in Banach spaces involving monotone operators, i.e. operators T from a Banach space X to its dual X^* for which

$$\operatorname{Re}\,(Tu - Tv, u - v) \geq 0 \tag{1}$$

for all u and v in X. A recent theorem of Zarantonello [18] for continuous bounded operators in Hilbert space obtains similar results for operators T satisfying the condition

$$|\,(Tu - Tv, u - v)\,| \geq c\,\|\,u - v\,\|^2. \tag{2}$$

In a preceding paper under the same title [14], the writer generalized and sharpened Zarantonello's result to obtain the following theorem:

THEOREM [14]. *Let X be a reflexive complex Banach space, X^* its dual, (w, u) the pairing between w in X^* and u in X. Let T be a mapping from X to X^* which is demicontinuous* [2] *(i.e. T is continuous from the strong topology of X to the weak topology of X^*). Suppose that T satisfies both of the following conditions:*

(i) *There exists a continuous real-valued function $c(r)$ on R^1 with $c(r) \to +\infty$ as $r \to +\infty$ such that*

$$|\,(Tu, u)\,| \geq c(\|\,u\,\|)\|\,u\,\| \tag{3}$$

for all u in X.

(ii) *For each $N > 0$, there exists a continuous increasing real-valued function $k_N(r)$ on R^1 with $k_N(0) = 0$ such that*

$$|\,(Tu - Tv, u - v)\,| \geq k_N(\|\,u - v\,\|)\|\,u - v\,\| \tag{4}$$

for all u and v in X with $\|\,u\,\| \leq N$, $\|\,v\,\| \leq N$.

Then T is a one-to-one mapping of X onto X^* and has a continuous inverse.

The serious part of the conclusion of this theorem, is of course, that the range of T is all of X^*.

In the present paper, it is our object to extend this result in two significant directions already considered by the writer in [1]–[13] for monotone operators.

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