OPERATOR REPRESENTATION THEOREMS[1]

BY

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We consider representations of bounded, compact and weakly compact linear operators from a Banach space to a space $BC(S)$, where S is an *arbitrary* topological space and $BC(S)$ is the space of bounded continuous scalar-valued functions on S with the sup norm. With the use of our theorems, one can quickly and easily deduce numerous operator representation theorems, many of which are new. For example, taking as domain space a space with a well-known conjugate space and range space as either c_0 or m, one fills in quite a few blanks in Tables VI A, B and C in [4]. Our proofs for range $BC(S)$, S arbitrary, are appreciably simpler than those found in the literature for range $C(S)$, S a compact Hausdorff space.

The spaces of bounded, compact and weakly compact linear maps from a B-space X to a B-space Y will be denoted, respectively, by $B[X, Y]$, $K[X, Y]$ and $W[X, Y]$. Unexplained terminology and notation will be found in [4].

Phillips [9] represented the general bounded operator from X to $B(S)$. Gelfand [5] represented the general bounded and compact operator from a B-space X to $C[0, 1]$ while Sirvint [11], [12] represented the general weakly compact operator. More recently, Bartle [1, Theorem 10.2] represented these three types of operators mapping X into the space $C(S)$ of continuous functions on a *compact Hausdorff* space S. However, Bartle's theorem is stated for $BC(S)$ with S an arbitrary topological space and it is wrong in this generality for compact and weakly compact maps. A counter-example is found by taking S to be an infinite set with the discrete topology (thus metrizable and locally compact), that is by taking the range to be $B(S)$. The compactness of S seems to be needed in the last part of the second sentence of the proof in [1].

A representation theorem for bounded operators from X into certain subspaces of $B(S)$ is given in [7]. Taylor [14] gives still more general representation theorems for bounded operators. In particular, he shows [13, Theorem 4.51-B] that Phillips' representation theorem is valid for the general bounded operator from X to $B(S, Z)$, where X and Z are Banach spaces and $B(S, Z)$ is the Banach space of bounded functions from the set S to Z with the sup norm. Wada [16] extended theorem 10.2 of [1] to the case where X is a barrelled locally convex space and $BC(S)$ is replaced by $C_K(S)$, where S is a completely regular Hausdorff space, K is a collection of compact sets which cover S, and $C_K(S)$ is the locally convex topological linear space of all real-valued continuous functions on S, equipped with the topology of uniform con-

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