GALOIS THEORY IN SEPARABLE ALGEBRAS OVER COMMUTATIVE RINGS

BY

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Introduction

In [1], M. Auslander and O. Goldman introduced the notion of a Galois extension of a commutative ring. The study of these Galois extensions was continued by S. Chase, D. K. Harrison and A. Rosenberg in [3] and by Harrison in [8]. Further work by Harrison [9] indicates that the notion of a Galois extension will have significant applications in the general theory of rings.

Throughout, K will denote a commutative ring (with 1) and S (with 1) a faithful K-algebra. Let G be a finite group of algebra automorphisms of S.

We call S a Galois extension of K with group G in case

1. $K = S^G$;
2. there exists $x_1, \cdots, x_n; y_1, \cdots, y_n \in S$ such that for all $a \in G$,

$$\sum_i x_i\, a(y_i) = 1 \quad \text{if} \quad a = e$$
$$= 0 \quad \text{if} \quad a \neq e$$

where if H is a subgroup of G, S^H denotes

$$\{x \in S \mid a(x) = x \quad \text{for all} \quad a \in H\}$$

This paper has as its purpose the study of not necessarily commutative Galois extensions of a commutative ring K. We show that if S is a Galois extension of K with no central idempotents except 0 and 1 then the center of S is left fixed by a normal subgroup of the Galois group. This reduces the study of Galois K-algebras S to the situation where S is either commutative or S is central over K. We concentrate here on the study of central Galois K-algebras whose Galois group is represented by inner automorphisms. The Galois group will always be represented by inner automorphisms in case K is a principal ideal domain, local ring, or field. We show that any central Galois K-algebra S whose Galois group G is represented by inner automorphisms is a separable projective group algebra. In case K has no idempotents but 0 and 1 we employ this result to find all the central Galois K-algebras with an Abelian Galois group of inner automorphisms. We conclude with an application to the commutative theory by giving a Kummer type theorem for Abelian extensions when appropriate roots of unity are present.

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