FURTHER REMARKS ON NONLINEAR FUNCTIONAL EQUATIONS

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Introduction

In three preceding papers under a similar title, [5], [6], [7], the writer has studied mappings T from a reflexive complex Banach space X to its dual X^* which we shall call *complex-monotone*. If (w, u) is the sesquilinear pairing between w in X^* and u in X, we shall call T *complex-monotone* if it satisfies the two conditions:

(I) *For each positive integer N, there exists a continuous, strictly increasing real function c_N on R^1 with $c_N(0) = 0$ such that*

$$|(Tu - Tv, u - v)| \geq c_N(\| u - v \|) \tag{1}$$

for all u and v with $\| u \| \leq N$, $\| v \| \leq N$.

(II) *There exists a real function c on R^1 with $c(r) \to +\infty$ as $r \to +\infty$ such that for all u,*

$$|(Tu, u)| \geq c(\| u \|)\| u \|. \tag{2}$$

It is the object of the present paper to sharpen and extend these results in several significant respects.

In the first place, in [5], [6], and [7], we discussed operators of two types, either $T = T_0 + C$ or $T = L + T_0 + C$, where T_0 is a nonlinear operator continuous from the strong topology of X to the weak topology of X^*, (demicontinuous), C is a nonlinear completely continuous operator from X to X^*, and L is a closed densely defined linear operator from X to X^* such that L^* is the closure of its restriction to $D(L) \cap D(L^*)$. As compared with the best results in the theory of monotone operators from X to X^* where comparable assumptions are made on Re $(Tu - Tv, u - v)$ and Re (Tu, u), (cf. [9]), these classes of operators seem too narrow in at least two respects. The continuity requirement on T_0 ought to be reduced to the assumption that T_0 is continuous from finite-dimensional subspaces of X to the weak topology of X^*. In addition, the perturbing completely continuous operator C should be allowed to intertwine itself with T_0 in a suitable sense rather than be merely an additional summand.

In Section 1, we carry through this weakening of requirements to obtain the following results:

THEOREM 1. *Let T be a nonlinear complex-monotone mapping of the reflexive complex Banach space X into its dual space X^*. Suppose that T is continuous*

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