LOCAL TIME AS A DERIVATIVE OF OCCUPATION TIMES

BY

RICHARD J. GRIEGO[1]

1. Introduction

In a joint paper [1] Blumenthal and Getoor obtained local times for a large class of Markov processes by considering local time as an additive functional of a Markov process. Abstract representation theorems insure the existence of continuous additive functionals with prescribed potentials. By prescribing a certain potential Blumenthal and Getoor were thus able to obtain a continuous additive functional that they called local time. The connection between local time and occupation times was then made under Hunt's hypothesis (F), [5, III]. Thus this method of obtaining local time is an indirect one.

It is of interest whether local time can be constructed for processes satisfying hypothesis (F) and certain regularity conditions by more direct and intuitive methods than those employed by Blumenthal and Getoor. In this paper local time is constructed as the limit (in some sense) of natural approximating densities.

2. Preliminaries

We refer the reader to Getoor's expository paper [3] for notation, definitions and results used below concerning Hunt processes and additive functionals.

Let $X = \{X_t, t \geqq 0\}$ be a Hunt process on a state space E. E is assumed to be a locally compact separable metric space with a point Δ adjoined to E as the point at infinity if E is not compact or an isolated point if E is compact. By convention all extended real valued functions on E are defined on $E \cup \{\Delta\}$ by $f(\Delta) = 0$. We denote the λ-potential operator of the process for $\lambda \geqq 0$ by U^λ, i.e.,

$$U^\lambda f(x) = E_x \int_0^\infty e^{-\lambda t} f(X_t)\, dt$$

where f is a bounded real-valued universally measurable function on E. Recall that if a Hunt process satisfies hypothesis (F) [5, III, p. 154] then there exists a measure ξ on E and point kernels $U^\lambda(x, y)$ defined on $E \times E$ for $\lambda \geqq 0$ such that

$$U^\lambda f(x) = \int U^\lambda(x, y) f(y)\, d\xi(y). \tag{2.1}$$

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