THE COHOMOLOGY OF STABLE TWO STAGE POSTNIKOV SYSTEMS[1]

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In this note we will study the cohomology algebra of a certain class of two stage Postnikov system. This question was considered originally in [4] [5] [6] [7]. We begin with a few definitions.

DEFINITION. By a generalized Eilenberg-MacLane space (GEM) we shall mean a Cartesian product of $K(\pi, n)$ spaces where π is a finitely generated abelian group and $n \geq 1$.

DEFINITION. A two stage Postinikov system $\mathcal{E}$ is a diagram

$$\begin{array}{ccc} F & = & F \\ \downarrow & & \downarrow \\ E & \longrightarrow & E_F \\ {\scriptstyle p}\downarrow & & \downarrow \\ B & \xrightarrow{\varphi} & B_F \end{array}$$

where

(i) F and B are GEM's.

(ii) $F \to E_F \xrightarrow{p} B_F$ is the path space fibration over B_F. B_F is of course a simply connected GEM.

(iii) $F \to E \to B$ is the fibre space induced from $F \to E_F \to B_F$ by the map $\varphi : B \to B_F$.

Now it is well known that B and B_F have H-space structures, unique up to homotopy, derived from the product in π. (In fact well chosen models are actually topological abelian groups.)

DEFINITION. $\mathcal{E}$ is called stable if B and B_F have H-space structures, multiplicatively homotopy equivalent to the standard ones, in which $\varphi : B \to B_F$ is a map of H-spaces.

Associated with a two stage Postnikov system we have an Eilenberg-Moore spectral sequence (see [12]) $\{E_r, d_r\}$ such that

$$E_r \Rightarrow H^*(E; k), \qquad E_2 = \mathrm{Tor}_{H^*(B_F;k)}(H^*(B; k), k)$$

where k is a field.

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