ON THE MULTIPLICATIVE EXTENSION PROPERTY II

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1. Introduction

Let B be a commutative Banach algebra. A subspace M of B has the multiplicative extension property (m.e.p.) if every linear functional on M of norm at most one is the restriction to M of a multiplicative linear functional on B. (See [1], [2] and [6].) Thus a subspace with the m.e.p. has the property that its conjugate space consists of scalar multiples of multiplicative functionals. This suggests the following generalization.

DEFINITION. A subspace M of B has the generalized m.e.p. if for some $\alpha > 0$, every linear functional on M of norm at most α is the restriction to M of a multiplicative linear functional on B. (In this case we say M has the α-m.e.p. If $\alpha = 1$, we just say M has the m.e.p.)

In [1] we considered certain examples of subspaces with the m.e.p. The purpose of this paper is to determine circumstances under which there exist subspaces with the generalized m.e.p. The basic general result is Theorem 2.2 which gives necessary and sufficient conditions for the existence of such a subspace. Theorem 2.3 gives a sufficient condition that we have found useful in construction of examples. These conditions were inspired by the construction in [1] of a subspace of the disc algebra with the m.e.p.

In Section 3 we investigate the generalized m.e.p. in function algebras on compact metric spaces and in the algebras $L^1(G)$, $M(G)$, and H^∞. We give an example of such a subspace of $M(G)$ that is different in nature from the original example of Hewitt and Kakutani [2].

The following notation is used throughout this paper. D is the open unit disc, Ω is the closed unit disc, and I is an index set. Ω^I denotes the functions on I into Ω with the product topology, and π_i is the projection of Ω^I into its i^{th} coordinate. δ_i is the element of Ω^I with $\delta_i(j) = 0$ if $i \neq j$ and $\delta_i(i) = 1$. If F is a subset of a Banach space E, c.l.s. F denotes the closed linear span of F.

We wish to thank the referee for pointing out an error in our original statement and proof of Theorem 2.2.

2. Generalities

Throughout this section B is a commutative Banach algebra with maximal ideal space $\mathfrak{M}_B = \mathfrak{M}$. We begin with a simple lemma giving conditions under which the generalized m.e.p. is preserved under isomorphisms.

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