ON THE STABLE HOMOTOPY GROUPS AND THE STABLE MOD-2 HOMOTOPY GROUPS OF Z_2-MOORE SPACES

BY

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Introduction

For each simply connected, reduced s.s. complex X, D. M. Kan [4] has defined as s.s. free group GX which serves as a loop complex for X. The lower central series $\{\Gamma_r GX\}$ of GX was studied by E. B. Curtis [1] who proved that the associated spectral sequence converges to the homotopy groups of X.

Here we confine ourselves to the Z_2-Moore spaces. In the following we shall derive a spectral sequence which converges to the stable mod-2 homotopy groups of Z_2-Moore spaces from the Curtis spectral sequence. An algebra structure is introduced in the E^1 term with the multiplication defined by composition of maps. We carefully study the derivations of the algebra E^1 and calculate the E^2 term in low dimensions. It is found that all the part of the E^2 term with dimensions ≤ 7 survives in the E^∞ term. Henceforth the stable mod-2 homotopy groups of Z_2-Moore spaces in dimensions ≤ 7 are obtained. Through the universal coefficient theorem and the stable version of the Blakers-Massey theorem applied to the cofibration

$$S^q \to S^q U_2 e^{q+1} \to S^{q+1},$$

the structure of the stable homotopy groups of Z_2-Moore spaces with dimensions ≤ 7 follows very easily.

1. Preliminaries

Since the statements in the following sections will be in terms of s.s. Lie rings, we recall some definitions and fundamental theorems which will be used later.

1.1. An s.s. complex X is a sequence of sets X_n for $n \geq 0$, with face operators $d_i : X_n \to X_{n-1}$ and degeneracy operators $s_i : X_n \to X_{n+1}$, $0 \leq i \leq n$ which satisfy the usual identities [4, p. 283]. If all the X_n and the d_i, s_i are objects and morphisms in a category C, X will be called an s.s. object over C.

THEOREM 1.2. *Let A, B be s.s. abelian groups and*

$$f_0, f_1 : A \to B$$

be s.s. homomorphisms. Then f_0, f_1 are homotopic if and only if Nf_0 and Nf_1 are chain homotopic, where N is the normalization functor defined by

$$(NG)_n = \textstyle\bigcap_{i=1}^{n} \ker d_i \approx G_n/DG_{n-1},$$

DG_{n-1} is generated by $s_i G_{n-1}$ for $0 \leq i \leq n-1$.

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