ON THE COHOMOLOGY OF THE MOD-2 STEENROD ALGEBRA AND THE NON-EXISTENCE OF ELEMENTS OF HOPF INVARIANT ONE

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A very handy E^1 term of the Adams spectral sequence for the sphere spectrum is obtained in [5]. Here we shall use it to calculate the cohomology of the mod-2 Steenrod algebra $H^{s,t}(A)$ in the range $s \leq 3$ and find some relations among the h_i's and c_j's in the range $s \leq 4$. The structure of $H^{3,t}(A)$ and the relations $h_0 h_i^3 \neq 0$ for $i \geq 4$ yield the information $d^2 h_i = h_0 h_{i-1}^2$ for $i \geq 4$ by an easy induction starting from $d^2 h_4 = h_0 h_3^2$. Hence a new proof for the non-existence of the elements of Hopf invariant one is obtained.

1. The structure of (E^1, d^1)

We recall from [5] that the structure of (E^1, d^1) is given by the following

THEOREM 1.1 [5]. (i) *(E^1, d^1) is a graded associated differential algebra (with unit) over Z_2 with*

(ii) *a generator λ_i (of degree i) for every integer $i \geq 0$*

(iii) *for every $m \geq 1$ and $n \geq 0$ a relation*

$$(1.2)_{n,m} \qquad \sum_{i+j=n} \binom{i+j}{i} \lambda_{i-1+m} \lambda_{j-1+2n} = 0$$

(iv) *d^1 is given by*

$$(1.3) \qquad d^1 \lambda_{n-1} = \sum_{i+j=n} \binom{i+j}{j} \lambda_{i-1} \lambda_{j-1}, \qquad n \geq 2.$$

Given a sequence of non-negative integers $I = (n_1, \cdots, n_r)$, we call r the length, $\sum_{i=1}^{r} n_i$ the degree, n_1 the leading integer and n_r the ending integer of I. I is said to be admissible if $2n_i \geq n_{i+1}$ for $1 \leq i \leq r-1$. Let λ_I stand for $\lambda_{n_1} \cdots \lambda_{n_r}$; then the additive structure of E^1 is given by the following

THEOREM 1.4 [5]. *The set consisting of the unit and λ_I with I admissible forms a vector basis for E^1.*

In the sequel we shall always express elements in E^1 in admissible forms, i.e., in terms of the above basis. Formulas (1.2) and (1.3) are written in symmetric forms. For the convenience of computation, it would be better to derive admissible expressions for them.

1.5. *The* mod-2 *binomial relations generated by $\lambda_i \lambda_{2i+1} = 0$ for $i \geq 0$.*

PROPOSITION 1.5.1. *There is a derivation $D : E^1 \to E^1$ sending λ_i to λ_{i+1} for $i \geq 0$.*

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