THE HOMOTOPY CATEGORY OF SPECTRA. I

BY

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The objective of this paper is to show that the homotopy category of semisimplicial spectra $\bar{\mathcal{S}}p_E$ in the sense of Kan [4] can be fully embedded in a very convenient manner into an abelian category $\mathcal{A}\bar{\mathcal{S}}p_E$ (Theorem 6.1). We mean by this the following: $\bar{\mathcal{S}}p_E$ coincides with the full subcategory of projectives of $\mathcal{A}\bar{\mathcal{S}}p_E$, $\mathcal{A}\bar{\mathcal{S}}p_E$ has enough injectives and projectives and the injectives and projectives coincide, and there exists a one-to-one correspondence between exact functors on $\mathcal{A}\bar{\mathcal{S}}p_E$ to an abelian category and functors on $\bar{\mathcal{S}}p_E$ to the same category which transform mapping cone sequences into exact sequences. Peter Freyd has proved [7] a general theorem according to which there exists for an additive category satisfying certain conditions a full embedding into an abelian category having properties of the above type; he has applied this to the stable category. It is the work of Freyd which suggested to the authors the considerations of the present paper.

In a letter to a friend of the authors R. L. Knighten stated that he knew some of the results below.

The point of view developed here facilitates the study of some questions concerning the structure of the homotopy category of spectra, such as the Postnikov resolutions and others, which will be dealt with in a subsequent paper. We believe that the homotopy category of spectra is important since it permits the classification of additive generalized cohomology theories.

In §1 we have collected for the convenience of the reader a few notions and results due to D. Kan and G. W. Whitehead, and we have adapted their covering homotopy theorem to our needs.

The main result is contained in Theorem 6.1, and the rest of the paper is devoted to setting up the machinery we need to prove this theorem.

The results contained in this paper have been announced in [1].

1. Preliminaries

The category of semisimplicial spectra $\mathcal{S}p$. The objects are semisimplicial spectra defined as follows: A semisimplicial spectrum X consists of

(i) for *every* integer q a set $X_{(q)}$ with a distinguished element $*$ (called *base point*); the elements of $X_{(q)}$ will be called *simplices* of *degree* q,

(ii) for every integer q and every integer $i \geq 0$ a function

$$d_i : X_{(q)} \to X_{(q-1)}$$

such that $d_i * = *$ (the *i-face* operator), and a function

$$s_i : X_{(q)} \to X_{(q+1)}$$

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