ON QUASI PROJECTIVES

BY

L. E. T. WU AND J. P. JANS[1]

1. Introduction

An R module M is said to be quasi injective if every homomorphism

$$T \xrightarrow{f} M,$$

where T is a submodule of M, can be extended to an endomorphism of M. See [5], [6], [10] for properties and applications of quasi injective modules. Phrased in terms of diagrams, a module M is quasi injective if every diagram

$$\begin{array}{ccccc} 0 & \longrightarrow & T & \xrightarrow{j} & M \\ & & \downarrow{\scriptstyle f} & & \\ & & M & & \end{array}$$

can be embedded in a commutative diagram

$$\begin{array}{ccccc} 0 & \longrightarrow & T & \xrightarrow{j} & M \\ & & \downarrow{\scriptstyle f} & \swarrow{\scriptstyle f'} & \\ & & M & & \end{array}$$

where j is the natural injection of T into M.

In this paper we shall be concerned with a concept dual to quasi injectives. A module M is said to be quasi projective if every diagram

$$\begin{array}{ccccc} & & M & & \\ & & \downarrow{\scriptstyle f} & & \\ M & \xrightarrow{n} & M/T & \longrightarrow & 0 \end{array}$$

can be embedded in a commutative diagram

$$\begin{array}{ccccc} & & M & & \\ & \swarrow{\scriptstyle f'} & \downarrow{\scriptstyle f} & & \\ M & \xrightarrow{n} & M/T & \longrightarrow & 0 \end{array}$$

where n is the natural map of M on M/T.

From the duality of the definitions of quasi projective and quasi injective, it is easy to deduce a number of properties of quasi projectives from the dual

Received June 27, 1966.

[1] Supported by a National Science Foundation grant.