TAME ORDERS, TAME RAMIFICATION AND GALOIS COHOMOLOGY

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Introduction

Let R be a commutative ring, and let G be a finite group, represented as R-automorphisms of a commutative R-algebra S. Consider the cohomology group $H^2(G, U(S))$, where $U(S)$ is the multiplicative group of units of S. To each cohomology class f, we associate a tower

$$R \subset S \subset \Delta(f, S, G)$$

of R-algebras, where $\Delta(f, S, G)$ is the crossed product algebra, or semi-linear group ring with factor set f. Classically, this tower has received considerable attention. For example, when S/R is a Galois extension of fields, or of rings, then $\Delta(f, S, G)$ is an R-central simple, or separable, algebra split by S, and defines an element of the Brauer group $B(S/R)$.

This is the case when S is an unramified extension of the integrally closed noetherian domain R, in a Galois extension L/K of their quotient fields. It is natural to consider the tower in a more general setting—for example, when the integral closure S of R in L is a tamely ramified extension of R. It is this case which is the focal point for the present investigations.

In particular, we consider the structure-forgetting functor from $\Lambda_f = \Delta(f, S, G)$-modules to S-modules. In our main theorem, we show that this functor preserves homological dimension for every cohomology class f if, and only if, the extension S/R is tamely ramified. The major corollary of this result indicates that all of the ramification in the tower $R \subset S \subset \Lambda_f$ takes place in the extension $R \subset S$.

A fortiori, a crossed product in a tamely ramified extension S/R is an order over R which is a reflexive R-module, and whose localization at every minimal prime ideal p of R is an hereditary order over R_p. Such an order is called a *tame order*. In Chapter I, we study the ideals and automorphisms of a tame order.

In Chapter II, we consider crossed product algebras and apply the theorems of Chapter I.

In Chapter III, we consider also Amitsur Cohomology, and applications to the study of the Brauer Group.

Although the main applications of these results are to integral extensions of integrally closed noetherian domains whose quotient field extension is finite

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