STRUCTURE OF MONOGENIC GROUPS

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Rubel [9] introduced the notion of a monogenic locally compact abelian group recently. This paper describes the structure of such groups. We introduce the notions of topologically divisible groups, canonical monogenic groups and amalgam of topological groups. We show that a totally disconnected monogenic locally compact abelian group is a direct product $L \times K$ where L is a topologically divisible group and K is either a compact monothetic group or a canonical monogenic group. If G is monogenic but not totally disconnected then it is either of the form $L \times K$ where L is topologically divisible and K is compact monothetic or is an amalgam $L + K$ of a compact monothetic group K and a group L such L/L_0 is topologically divisible where L_0 is the connected component of identity of L. The structure of topologically divisible groups and of canonical monogenic groups are described. In the totally disconnected case the structure is an exact generalization of the result of [7]. The notion of amalgam was discussed by B. H. Neumann for discrete groups.

Conventions and Notations. All groups occurring in this paper are assumed to be locally compact Hausdorff and abelian groups. All notions in abstract abelian groups are to be found in [3] and [4]. All notions in topological groups which are not defined here are to be found in [5] or [10]. $R^n (n \geq 0)$ denotes the usual real Euclidean group. If p is a prime then I_p^* denotes the group of p-adic integers, and J_p the group of p-adic numbers.

DEFINITION 1. Let G be a group. A compact character χ of G is a continuous character of G which is also open.

DEFINITION 2 (Rubel). A group G is called monogenic if there exists an element $x_0 \in G$ such that whenever H is a subgroup of G such that G/H is compact we have that $\varphi(x_0)$ generates G/H topologically where $\varphi : G \to G/H$ is the canonical map. Such an element x_0 is called a special element of G.

DEFINITION 3. A group G is called topologically divisible if the only compact character of G is the identity character. (See also [8]).

Note 4. The group J_p is topologically divisible for all primes p and every discrete divisible group is also topologically divisible. A group G is topologically divisible if and only if whenever H is a closed subgroup of G such that G/H is compact we have that $H = G$. Loosely speaking a group G is topologically divisible if and only if it admits no non-trivial compact quotient groups. In this sense our definition of topologically divisible groups generalizes the

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