ON THE EXTENSIONS OF THE INFINITE CYCLIC GROUP BY A 2-MANIFOLD GROUP

BY

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In this note we shall study certain group extensions

$$E : 0 \to F \xrightarrow{i} M \xrightarrow{\pi} B \to 1$$

where F is Abelian (written additively), M and B are non-Abelian (written multiplicatively) and all groups are assumed to have finite presentations. Following [2] we denote by $\varphi : B \to \operatorname{Aut} F$ "conjugation by elements of B" determining the B-module structure of F. A morphism $\Gamma : E \to E'$ is a triple $\Gamma = (f, g, h)$ of commuting homomorphisms:

$$(*) \qquad \begin{array}{ccccccccc} E : & 0 \to & F & \xrightarrow{i} & M & \xrightarrow{\pi} & B & \to 1 \\ & & f\downarrow & & g\downarrow & & h\downarrow & \\ E' : & 0 \to & F' & \xrightarrow{i'} & M' & \xrightarrow{\pi'} & B' & \to 1 \end{array}$$

The classical theory defines a *congruence* ($E \equiv E'$) as a morphism $\Gamma : E \to E'$ such that $F = F'$, $B = B'$ and $\Gamma = (1_F, g, 1_B)$. It follows that g is an isomorphism and $\varphi = \varphi'$. The main result is that for given φ the congruence classes are in one-to-one correspondence with $H^2_\varphi(B; F)$.

DEFINITION. An *equivalence* of extensions ($E \sim E'$) is a morphism $\Gamma : E \to E'$ where f and h are isomorphisms.

For convenience we shall assume $F = F'$, $B = B'$ and supress i and π. The non-commutative 5-lemma implies that g is an isomorphism. The following are standard or easily verified:

PROPOSITION 1.

(i) *"$\sim$" is an equivalence relation*
(ii) $E \equiv E' \Rightarrow E \sim E'$.
(iii) $E \sim E'$ *gives rise to a commutative diagram*

$$(**) \qquad \begin{array}{ccc} B & \xrightarrow{\varphi} & \operatorname{Aut} F \\ h\downarrow\approx & & \downarrow\approx f^* \\ B & \xrightarrow{\varphi'} & \operatorname{Aut} F \end{array}$$

where f^* is conjugation by f.

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