SIMPLE TESTS FOR RECURRENCE OR TRANSIENCE OF INFINITE SETS IN RANDOM WALKS ON GROUPS

BY

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1. Introduction

It has been shown by R. A. Doney [3] that in the case of the simple three-dimensional random walk no condition of the type

$$\text{I} \qquad \sum_{a \in A} \phi(a) = \infty$$

with $\phi(a) \geq 0$ can be necessary and sufficient for a set A to be recurrent.

In this paper the analogous result is obtained for an arbitrary transient random walk on an Abelian group provided only that $G_{0i} \to 0$ as $i \to \infty$.

In what follows we will use the terminology and also some of the results of [6]. We assume that a countable group G is given with its elements numbered in some order $e = a_0, a_1, a_2, \cdots$. By a random walk on G we mean a Markov chain for which the probabilities

$$(1.1) \quad p_{ij}^{(n)} = \Pr(x_{m+n} = a_j : x_m = a_i) = \Pr(x_n = a_i^{-1} a_j : x_0 = e)$$

are functions of $a_i^{-1} a_j$, n, x_n denoting the element of G reached by the random walk at time n.

We also write

$$(1.2) \quad \begin{aligned} e(a, A) &= \Pr(x_n \notin A \quad \text{for } n > 0 : x_0 = a) \\ f(a, A) &= \Pr(x_n \in A \quad \text{for some} \quad n \geq 0 : x_0 = a) \\ f_{ij} &= f(a_i, \{a_j\}) \qquad (i, j \geq 0) \\ G_{ij} &= \sum_{n=0}^{\infty} p_{ij}^{(n)} = f_{ij} G_{jj} = f_{ij} G_{00} \qquad (i, j \geq 0) \end{aligned}$$

where G_{00}, and hence also each G_{ij}, is finite for a transient random walk. We say that a set A in G is recurrent if $f(a, A) = 1$ for all a in G, or equivalently,

$$(1.3) \quad \begin{aligned} 1 &= h(a, A) \\ &= \Pr(x_n \in A \quad \text{for infinitely many} \quad n \geq 0 : x_0 = a) \quad (a \in G). \end{aligned}$$

A is said to be transient if $h(a, A) = 0$ for all a in G.

A set C in G is said to be almost closed if

$$(1.4) \qquad 0 \lneqq h(a, C) = 1 - h(a, G - C) \qquad (a \in G).$$

An almost closed set C is atomic if it does not contain two disjoint almost

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