ON THE TRIANGULATION OF THE REALIZATION OF A SEMISIMPLICIAL COMPLEX

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In some mimeographed notes of Barratt [1], in which he shows that the geometric realization of a semisimplicial complex has a simplicial subdivision, there appear to be several errors, one in the statement of the subdivision theorem and two more in the proof. This note will give (we hope) a correct statement and proof of this theorem (Theorem 1.1), and will draw as a consequence the theorem of Milnor [4] that the homotopy groups of the realization $| S(X) |$ of the singular complex of a space are naturally isomorphic to those of the space itself.

We will write ssc for semisimplicial complex. Notation and terminology as in [4] or [5], except that we will denote the abstract n-simplex by $\Sigma(n)$, the geometric n-simplex by Δ^n.

The main result is the following theorem.

Theorem 1.1. *Let X be a ssc and $| X |$ its realization* [4]. *Then there is a functor D from the category of ssc's to that of ordered simplicial complexes, a transformation of functors $\lambda : D \to 1$, and, for each X, a map $t_x : | DX | \to | X |$ such that*

(i) *t_x is a homeomorphism (and therefore a triangulation of $| X |$;*

(ii) *t_x defines a subdivision of the* CW *complex $| X |$; and*

(iii) *$| \lambda(X) |$ is homotopic to t_x by a homotopy F such that for each cell $| e |$ of $| DX |$, F maps $| e | \times | I$ into the smallest cell $| x |$ of $| X |$ which contains $t_x(| e |)$.*

We will give the proof later, in Sections 2, 3, and 4.

Corollary 1.2. (Simplicial approximation theorem). *If $f : | X | \to | Y |$ is any map, then f is homotopic to the realization of a ss map of subdivisions of $| X |$ and $| Y |$. If $| X |$, $| Y |$ are finite and therefore metrizable, then for any prescribed $\varepsilon > 0$, the homotopy between f and its ss approximation can be chosen so that it does not displace a point outside of an ε-disc.*

Proof. Apply the simplicial approximation theorem to the map

$$f' : | DX | \to | DY |, \quad \text{where} \quad f' = t_y^{-1} f t_x . \quad \blacksquare$$

Some remarks about the singular complex of a space. Let X be a space, and $S(X)$ its singular complex. Let $p_X : | S(X) | \to X$ be the map sending the point (P, x_n) of $| S(X) |$ into $x_n(P)$ [4].

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