FULL REFLECTIVE SUBCATEGORIES AND GENERALIZED COVERING SPACES

BY

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Introduction

Let $\mathcal{A}$ be a full subcategory of $\mathcal{C}$. A morphism $e : X \to X^*$ is said to be a *reflection map* which reflects X into the category $\mathcal{A}$ iff $X^* \in \mathcal{A}$ and every morphism $f : X \to A$ with $A \in \mathcal{A}$ can be factored as $f = ge$ for a unique $g : X^* \to A$. The full subcategory $\mathcal{A} \subseteq \mathcal{C}$ is *reflective* if every object $X \in \mathcal{C}$ admits a reflection map $e : X \to X^*$. Dually $\mathcal{A} \subseteq \mathcal{C}$ is *coreflective* if each $X \in \mathcal{C}$ admits a *coreflection map* $e : X^* \to X$ such that $X^* \in \mathcal{A}$ and each morphism $f : A \to X$ with $A \in \mathcal{A}$ factors as $f = eg$ for a unique g.

In this paper we obtain necessary and sufficient conditions for a full subcategory to be reflective. Our methods also yield some information about the reflection maps and the full reflective subcategory generated by certain types of subcategories. Applying the dual of these results to the category of pointed topological spaces (i.e. topological spaces with base points), we show that the simply connected spaces in the sense of Hu [4] generate an interesting coreflective subcategory. The coreflection maps, $p : (X^*, x^*) \to (X, x)$ can be regarded as generalized universal coverings since p is the usual universal covering if such a covering exists. In general X^* is simply connected in the sense of Chevalley [2] and p is a fiber map with pathwise totally disconnected fibers. (An example shows that the fibers are not always discrete.) X^* appears to be related to the *universal procovering* of X obtained by Lubkin [11] for certain spaces. Our coreflection enables us to extend some of the Lubkin and Chevalley theory of covering spaces to the category of **all** topological spaces.

As is the case with many conditions for reflectivity our conditions are closely related to the Freyd adjoint functor theorem [3 p. 84] even though a knowledge of that theorem is not a prerequisite for reading this paper. In effect, we show that for full reflective subcategories, the "solution set" hypothesis of the adjoint functor theorem can always be satisfied in a convenient, canonical way which leads to simplifications; these simplifications are illustrated by the well known type of theorem that for a well-behaved category $\mathcal{C}$, a full subcategory $\mathcal{A} \subseteq \mathcal{C}$ is reflective if $\mathcal{A}$ is closed under the formation of products and subobjects (e.g. see [3, p. 87]). Isbell in [5] and [6] demonstrates that the notion of "subobject" is often best defined in the context of a bicategory structure (defined below). In [5, p. 1276], it is shown that if $\mathcal{C}$ is well-behaved and has a suitable bicategory structure, then $\mathcal{A} \subseteq \mathcal{C}$ is reflective whenever $\mathcal{A}$ is closed under the formation of products and subobjects in the bicategory sense. These condi-

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