A PATHOLOGICAL FIBER SPACE[1]

BY

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1. Introduction

The following result is a consequence of the work of Dyer and Hamstrom [1]. Suppose K, X and Y are metric spaces, K compact X complete, the dimension of Y is equal to n and the space of homeomorphisms of K onto itself (c-o topology) is LC^{n+1}. Then if f is a completely regular mapping from X to Y such that the inverse of each point is homeomorphic to K, then f is locally trivial.

Since it is conjectured that the space of homeomorphisms of a manifold is locally connected in all dimensions, the above theorem gives rise to the question as to whether the local connectivity of the space of homeomorphisms could be replaced by assuming that K is a manifold or an absolute retract.

In [3] McAuley conjectured: Suppose that (E, p, B) is a Serre fibration and that E and B are finite-dimensional Peano continua. Then if each fiber is homeomorphic to a fixed Peano continuum, p is locally trivial.

In this paper an example is given which would answer the first question negatively for K an absolute retract and the example also shows that McAuley's conjecture is false even for Hurewicz fibrations.

2. Definitions

(2.1) A map p from a metric space E onto a metric space B is *completely regular* if given any $b \in B$ and any $\varepsilon > 0$ there exists $\delta > 0$ such that if $b_1 \in B$ and $d(b, b_1) < \delta$ then there exists a homeomorphism from $p^{-1}(b)$ onto $p^{-1}(b_1)$ which moves no point as much as ε.

(2.2) A map p from a space E onto a space B is a *Hurewicz fibration* if the mapping

$$p^* : E^I \to Z = \{(e, f) \in E \times B^I \mid p(e) = f(0)\}$$

defined by $p^*(g) = (g(0), pg)$ admits a section.

It should be noted that if p is a Hurewicz fibration then p has the absolute covering homotopy property.

(2.3) A mapping p from a space E onto a space B is *locally trivial* if there exists a space F such that for each $b \in B$, there is an open neighborhood U of

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[2] The author would like to express his appreciation to the referee whose suggestions considerably shortened the proofs of (3.3) and (3.5).