ON THE NUMBER OF CO-MULTIPLICATIONS OF A SUSPENSION[1]

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In [2], Arkowitz and Curjel established a criterion for determining when an associative H-space possesses only a finite number of multiplications. That their result can be dualized is the subject of the present note.

An H'-structure, or co-multiplication, on a space Z is a based map

$$\varphi : Z \to Z \vee Z$$

which has the property that the compositions $\pi_1 \circ \varphi$ and $\pi_2 \circ \varphi$ are both homotopic to the identity map of Z, where π_1 and π_2 are the obvious projections.

Let X be a CW-complex of locally-finite type. Then, by the Hilton-Milnor Theorem, $\Omega\Sigma(X \vee X)$ is homotopy equivalent to $\prod_k \Omega\Sigma P_k$ where k runs through a set of basic products for the set $\{1, 2\}$. To each basic product k there is associated a positive integer $\omega(k)$, the *weight* of k, and P_k has the homotopy type of

$$\overbrace{X \wedge X \wedge \cdots \wedge X}^{\omega(k)}.$$

Moreover, the homotopy equivalence is given by a map of the form $\prod_k \Omega g_k$, where $g_k : \Sigma P_k \to \Sigma(X \vee X)$ is an iterated generalized Whitehead product which is associated with the basic product k. In particular $P_1 = P_2 = X$ and the maps $g_i : \Sigma X \to \Sigma(X \vee X)$ $(i = 1, 2)$ are the inclusions. All g_k with $\omega(k) \geq 2$ are Whitehead products involving both the first and second factors of $\Sigma(X \vee X)$. For more details see [3] or [7].

If $f : X \to \Omega\Sigma(X \vee X)$ is any map, then there is a map $\bar{f} : X \to \prod_k \Omega\Sigma P_k$ with $\prod_k \Omega g_k \circ \bar{f} \sim f$. Let $p_k : \prod \Omega\Sigma P_k \to \Omega\Sigma P_k$ denote the projection, and let $\pi_i : \Sigma(X \vee X) \to \Sigma X$ $(i = 1, 2)$ denote the projections.

THEOREM 1. $\Omega\pi_i \circ f \sim p_i \circ \bar{f}$ $(i = 1, 2)$.

Proof. By the above, $\Omega\pi_1 \circ f \sim \Omega\pi_1 \circ \prod_k \Omega g_k \circ \bar{f}$. Since $\Omega\pi_1$ is a homomorphism, $\Omega\pi \circ \prod_k \Omega g_k = \prod \Omega(\pi_1 \circ g_k)$. But every basic product k with $\omega(k) \geq 2$ involves both 1 and 2, thus $\pi_i \circ g_k \sim *$ $(i = 1, 2$ and $\omega(k) \geq 2)$. Since also $\pi_1 \circ g_2 \sim *$ and $\pi_2 \circ g_1 \sim *$, Theorem 1 is proved.

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