INVARIANT IDEALS OF POSITIVE OPERATORS IN $C(X)$. II

BY

H. H. SCHAEFER[1]

The present paper constitutes the second part of a study of ideals in $C(X)$ invariant under a given positive linear operator. While it will be necessary to have part I[2] on hand for an understanding of certain details, we shall briefly recall some basic definitions, notations, and results of part I.

Throughout the paper, T denotes a positive linear operator on the complex Banach algebra $C(X)$, where X is compact Hausdorff. A T-ideal (Def. 1) is a closed proper ideal $J \subset C(X)$ such that $T(J) \subset J$. Every T-ideal J gives rise to a positive operator T_J on $C(X)/J$. In general, $C(X)/J$ is identified with $C(S_J)$, where S_J (called the support of J) is the unique closed subset of X such that $J = \{f : f(S_J) = (0)\}$. T is called irreducible (Def. 2) if (0) is the only T-ideal; a T-ideal J is maximal if and only if T_J is irreducible. T is called ergodic (Def. 3) if for each $f \in C(X)$, the convex closure of the orbit $\{f, Tf, T^2f, \cdots\}$ contains a fixed vector of T; if the semigroup $\{T^n\}$ is bounded, ergodicity of T is equivalent with the strong convergence (for $n \to \infty$) of the averages

$$M_n f = n^{-1}(f + Tf + \cdots + T^{n-1}f) \to Pf,$$

P being a positive projection onto the fixed space of T. If $M_n \to P$ norm converges, T is called uniformly ergodic (Def. 3a). If $Te = e$ where $e(s) = 1$ for all $s \in X$, T is called a Markov operator.

THEOREM 1 (§2). *For each maximal T-ideal J, there exists an eigenvector (measure) $\phi \geq 0$ of the adjoint operator T' such that*

$$J = \{f : \phi(\,|f|\,) = 0\}.$$

The corresponding eigenvalue $\rho \geq 0$ is zero iff ϕ is supported by a single point $s \in X$ for which $Te(s) = 0$.

THEOREM 2 (§3). *If T is an ergodic Markov operator and Φ denotes the (weak* compact) set of all positive, normalized T-invariant measures on X, the mapping*

$$\phi \to I_\phi = \{f : \phi(\,|f|\,) = 0\}$$

is a bijection of the set Λ of extreme points of Φ onto the set of all maximal T-ideals. Moreover, every T-ideal I_ϕ ($\phi \in \Phi$) is the intersection of all maximal T-ideals containing it.

Received October 23, 1967.

[1] Supported in part by a National Science Foundation grant.

[2] Illinois J. Math, vol. 11(1967), pp. 703–715. Numeration of definitions, results and references is continued from part I.