HOMOTOPY NORMAL BUNDLES FOR LOCALLY FLAT IMMERSIONS AND EMBEDDINGS OF TOPOLOGICAL MANIFOLDS

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1. Introduction

Let M denote a topological n-manifold and N a topological $(n + k)$-manifold. For locally flat embeddings or immersions, $f: M \to N$, Fadell [2], [3] has constructed normal fiber spaces which satisfy a weak Whiteny duality theorem. Each of these normal fiber spaces is an appropriate path space similar to that described by Nash [12]. In Fadell's construction, the case of immersions is treated separately from that of embeddings and is, in fact, much more complicated. Indeed, in both cases the fibers are rather messy topological spaces.

Our objective is to exploit the theory of microbundle pairs and the corresponding version of the Kister-Mazur coring theorem, proven independently by Kuiper and Lashof [7] and the author [9], to obtain a simpler normal structure, the homotopy normal bundle of f. It will be shown that this h-normal bundle satisfies a weak Whitney duality (Theorem 4.5), a composition theorem (Theorem 4.7), and an appropriate "isotopy invariance" theorem (Theorem 4.9). Furthermore, we shall prove that if f has a tubular neighborhood, ν, then ν is fiber homotopy equivalent to the h-normal bundle (Theorem 4.11) and hence tubular neighborhoods are unique up to fiber homotopy equivalence. Finally we shall prove that the h-normal bundle is equivalent, in so far as is possible, to the normal structures previously given in both the smooth and topological categories.

In Section 2 we give a rather general form of the coring theorem which, with the results of Section 3 on Euclidean bundles, allows us, in Section 4, to define the h-normal bundle and verify its properties.

2. The coring theorem

The Kister-Mazur theorem [6] has been generalized by Kuiper and Lashof [7] and, in the topological case, by the author [9] as follows:

THEOREM 2.1. *In the topological and in the PL-category every* (R^{n+k}, R^n)-*microbundle,* (a, b), *over a locally finite simplicial complex contains a unique* (R^{n+k}, R^n)-*bundle,* (α, β), *in the sense of Steenrod* [12], *which is* (R^{n+k}, R^n)-*microbundle equivalent to* (a, b).

The coring theorem has been extended further, in [4] and [9], by weakening

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