ALGEBRAIC GROUPS AND HOPF ALGEBRAS

BY

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1. Introduction

We consider affine algebraic groups over a fixed algebraically closed base field of characteristic 0. The totality of all polynomial functions on such a group is the underlying set of a Hopf algebra over the base field, and the group is recoverable from this Hopf algebra structure. Similarly, a Hopf algebra is attached to a Lie algebra. The elements of this Hopf algebra are the representative functions on the universal enveloping algebra of the Lie algebra.

The main theme of this paper is the comparison of the Hopf algebra of an algebraic group with the Hopf algebra of its Lie algebra. Within the theory of algebraic groups, this theme is very closely tied to that of group coverings. In particular, we shall exhibit the use of Hopf algebras in constructing universal coverings in the category of affine algebraic groups. An affine algebraic group has such a universal covering if and only if its radical is unipotent. Our procedure also yields a direct description of the "simply connected" affine algebraic group belonging to a given Lie algebra L such that $L = [L, L]$ in terms of the universal enveloping algebra of L.

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2. Algebraic group coverings

Let F be an algebraically closed field of characteristic 0, and let G and H be connected affine algebraic groups over F. Suppose that $\eta : H \to G$ is a surjective rational group homomorphism with finite kernel. Then we say that (H, η) is a *group covering* of G. We call G *simply connected* if every group covering of G is an isomorphism.

LEMMA 2.1. *Suppose that G is simply connected, and that K is a connected normal algebraic subgroup of G. Then G/K is simply connected.*

Proof. Let γ denote the canonical epimorphism $G \to G/K$, and consider a group covering $\eta : H \to G/K$. We must show that the kernel, A say, of η is trivial. Let us form the fibered product $P = H \times_{(\eta,\gamma)} G$, i.e., the algebraic subgroup of $H \times G$ consisting of all elements (h, g) such that $\eta(h) = \gamma(g)$. Clearly, K may be identified with a normal algebraic subgroup of P, and P/K is isomorphic with H. Since H and K are connected, it follows that P is connected. Now consider the canonical projection epimorphism $\rho : P \to G$. The kernel of ρ evidently coincides with the canonical isomorphic image of A

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