Q-SIMPLICIAL SPACES[1, 2, 3]

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The quasicomplexes of Lefschetz have proven useful in investigating the fixed point properties of exotic spaces [1], [3], [11]. The purpose of this paper is to report a simplification of Leftschetz's quasicomplexes which is used to give a unified treatment of several forms of the Lefschetz fixed point theorem, including that part of the Schauder-Leray theory to which the Leftschetz theorem is pertinent. No attempt is made to consider ramifications of the related concept of the local degree theory of Leray, for my spaces will not always be locally connected.

In this study, I adopt the point of view apparently taken by Lefschetz [5] when he defined quasicomplexes: the key step in the classical proof of the Lefschetz theorem for simplicial complexes involves the subdivision operator, and if one is to extend the classical result he could do so by looking for a substitute for this step. The result of such an effort is the concept of "Q-simplicial spaces". Here the Q stands not for "quasi" but for the coefficient field of which the Lefschetz number is a member. Q-simplicial spaces enjoy some advantages over the quasicomplexes defined by Lefschetz [5], and the weak semi-complexes of Thompson [13]. Their definition is syntactically simpler. They are not necessarily compact: a locally convex topological vector space is Q-simplicial. Open subsets of, retracts of, and infinite products of Q-simplicial spaces are all again Q-simplicial.

The principal technique used to derive these properties uses the fact that the *category* of Q-simplicial spaces and continuous maps has a certain class of infinite limits. My formulation of this class of limits is not categorical, however, it being simpler and more direct to define these limits in topological terms. The formulation is more in keeping with the concept, introduced by Klee [8], of "approachable sets," and so I have adopted that terminology for describing my limits.

The proof of the Lefschetz theorem itself for Q-simplicial spaces is conceptually simplified in that no reference to chain homotopies is necessary; instead, use is made of the continuity axiom for Čech theory.

The last section is concerned with examples which show (1) Q-simplicial spaces do not have a local characterization, (2) the class of Q-simplicial spaces depends in an essential way upon the characteristic of Q.

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