# ON THE FIRST MAIN THEOREM ON BLOCKS OF CHARACTERS OF FINITE GROUPS

BY

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## 1. Introduction[1]

Let $G$ be a finite group and let $p$ be a fixed prime number. The first main theorem on blocks establishes a one-to-one correspondence between the $p$-blocks $B$ of $G$ with the defect groups $D$ and the $p$-blocks $b$ of the normalizer $N_G(D)$ of $D$ with the defect group $D$, cf. [2], [3]. It is the purpose of this note to show that this theorem can be derived easily from the results of [4]. We shall need only the results (2A), (2B), (3A), (3B) and (4A) of [4]. In particular, we shall not need any results dealing with fields of characteristic 0. A proof of the main theorem on blocks operating completely within a fixed field $\Omega$ of characteristic $p$ has already been given by A. Rosenberg [5].

We use the same notation as in [4]. In particular, $\Omega$ will denote an algebraically closed field of characteristic $p$, $\Omega[G]$ will denote the group algebra of $G$ over $\Omega$, and $Z = Z(G)$ will be the class algebra of $G$ over $\Omega$ (i.e., $Z(G)$ is the center of $\Omega[G]$). As remarked in [4], the results (3A), (3B), and (4A) of [4] remain valid, if instead of the decomposition of $Z(G)$ into block ideals we consider more generally any decomposition

$$Z = A_1 \oplus A_2 \oplus \cdots \oplus A_r \tag{1}$$

as a direct sum of ideals $A_i$. Each $A_i$ is a direct sum of block ideals of $Z$. Let $\hat{Z}$ denote the dual space consisting of all linear functions defined on $Z$ with values in $\Omega$.

An ideal $A \neq (0)$ occurs as a summand in a decomposition (1), if and only if $A$ has the form $\eta_A Z$ where $\eta_A$ is an idempotent of $Z$. We may consider the dual space $\hat{A}$ of $A$ as a subspace of $Z$ by extending each $f \in \hat{A}$ linearly so that it vanishes on the complement $(1 - \eta_A)A$. (In [4], the notation $F_A$ was used for this subspace of $Z$.) If $Q$ is a $p$-subgroup of $G$, the multiplicity $m_A(Q)$ of $Q$ as a lower defect group of $A$ is defined as follows. Consider subspaces $V$ of $\hat{A}$ with the following two properties.

(i) For each $f \neq 0$ in $V$, there exists a conjugate class $K$ of $G$ with the defect group $Q$ such that $f(\mathcal{S}K) \neq 0$. (Here $\mathcal{S}K$ is the class sum of $K$.)

(ii) For $f \in V$, we have $f(\mathcal{S}K) = 0$ for all conjugate classes $K$ of $G$ whose defect group has lower order than $Q$.

Then $m_A(Q)$ is the maximal dimension of such $\Omega$-spaces $V$.

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