ON THE UNIQUENESS THEOREM

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It is the purpose of this note to give an alternate proof of the following theorem which originally is an intermediate result of [1].

THEOREM (Feit and Thompson). *Let G be a simple group of odd order all of whose proper subgroups are solvable. Let E be an elementary abelian p-subgroup of order p^3 in G. Then there is only one maximal subgroup of G which contains E.*

The largest part of the proof deals with the Fitting subgroup F of a maximal subgroup H of G. In §2 we consider the case that F is a p-group; necessary results about F are derived in a well known way mainly from the Transitivity Theorem (see (1.1) below) and the ZJ-Theorem (1.2). The case that F is not a p-group is treated in §3; here a very simple observation is crucial, namely that arguments in the proof of the Transitivity Theorem can be applied to certain subgroups of F.

In §4, knowledge about F is used to obtain information about subgroups of H not necessarily contained in F. Finally transfer arguments finish the proof of the theorem.

In the remainder of this section we introduce some notation and collect some necessary lemmas.

Notation.

S_p-subgroup = Sylow p-subgroup
$X^{\#}$ = set of non-identity elements of X
$F(X)$ = Fitting subgroup of X = maximal nilpotent normal subgroup of X
$J(P)$ = subgroup generated by all the abelian subgroups of maximal possible order of P
$\mathcal{N}_Y(A, \pi)$ = set of A-invariant π-subgroups of Y
$\mathcal{N}_Y^*(A, \pi)$ = set of maximal elements of $\mathcal{N}_Y(A, \pi)$
group of type $(p, p, \cdots, p)$ = elementary abelian p-group of order p^n
$r(X) \geq n$ means that X has an elementary abelian p-subgroup of order p^n
$SCN_n(P)$ = set of abelian normal subgroups of P satisfying $C_P(A) = A$ and $r(A) \geq n$
$\{\cdots\}$ = the set $\cdots$
$\langle\cdots\rangle$ = the subgroup generated by $\cdots$

In the following sections G is assumed to be a group of odd order all of whose proper subgroups are solvable.

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