ON MANIFOLDS WITH CONJUGATION

BY
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1. Introduction

The concept of a conjugation on an almost complex manifold M^{2n} was defined by Conner and Floyd [2, §24]. Specifically, an almost complex structure on M is a real linear bundle map $J : \tau_M \to \tau_M$ on the tangent bundle of M, covering the identity map of M, and such that $J^2 = -1$, and a conjugation on (M, J) is an involution $\sigma : M \to M$ whose differential $d\sigma : \tau_M \to \tau_M$ is a conjugate linear isomorphism (i.e. $d\sigma \cdot J = -J \cdot d\sigma$).

The object of this paper is to analyze the cobordism classification of such conjugations. One approach to this sort of problem occurs in Landweber [4], considering equivariant homotopy of the Thom spectrum MU. Lacking strong transverse regularity theorems, this homotopy question is not as closely related to the geometry of the problem as one would wish. The approach taken here is analogous to the geometric part of the Conner and Floyd study of involutions.

In order to perform a cobordism analysis, one must first enlarge the collection of objects under study to give a suitable boundary, since almost complex manifolds are always even dimensional. This is performed in Section 2 by defining the notion of a conjugation on a stably almost complex manifold (as studied by Milnor [7]). Another way to describe such a manifold structure is to be given a manifold M with involution σ and an isomorphism of the normal bundle of M with an Atiyah-real vector bundle over (M, σ) (see Atiyah [1]). If (M, σ, J) is a conjugation on an almost complex manifold, the inverse to the Atiyah-real bundle $(\tau_M, J, d\sigma)$ over (M, σ) provides a stably almost complex conjugation structure on M.

One may then form cobordism groups in the standard way, and the ring of cobordism classes of stably almost complex conjugations is denoted Ω_*^{AR}. By restricting to conjugations for which the underlying involution σ is fixed-point free, one may form the cobordism ring of free stably almost complex conjugations, denoted $\hat{\Omega}_*^{AR}$. In Section 3, the interrelationship of these rings is studied, making use of the relative cobordism group of conjugations on stably almost complex manifolds with free action on the boundary, denoted $\bar{\Omega}_*^{AR}$. One then has a rather obvious exact sequence

$$\hat{\Omega}_*^{AR} \xrightarrow{F} \Omega_*^{AR} \xrightarrow{i} \bar{\Omega}_*^{AR} \xrightarrow{\partial} \hat{\Omega}_*^{AR}$$

similar to the sequences of Conner and Floyd [3] or [2, §28.1]. Further, using the fixed-point method, one may analyze $\bar{\Omega}_*^{AR}$ and reduce this to the calculation

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