MULTIPLICATION ALTERATION BY TWO-COCYCLES

BY

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0. Introduction

If U is an associative unitary algebra with a commutative subalgebra A and $\sigma = \sum a_i \otimes b_i \otimes c_i \in A \otimes A \otimes A$ is an Amitsur 2-cocycle, then we can define a new multiplication $*$ on U by setting

0.1 $$u * v = \sum a_i u b_i v c_i$$

for all $u, v \in U$. The Amitsur 2-cocycle condition guarantees that U is associative and unitary with the $*$ multiplication. If U was originally a central separable (simple) algebra then U is still central separable under the new multiplication. We show that the central separable algebra resulting from an Amitsur 2-cocycle σ is isomorphic to the Rosenberg Zelinsky central separable algebra coming from the 2-cocycle σ^{-1}.

If K is an intermediate field ($A \supset K \supset k$) we show how mapping 2-cocycles in $A \otimes_k A \otimes_k A$ into $A \otimes_K A \otimes_K A$ corresponds to taking the centralizer of K in central separable k algebras with maximal commutative subfield A. On the way to these results we prove that if A is a finite purely inseparable field extension of k and U is an algebra containing A then U is isomorphic to $A \otimes_k A$ as an A-bimodule if and only if U is a central separable k algebra of k-dimension n^2.

By being careful about what we mean by a 2-cocycle we are able to obtain an associative unitary algebra by means of 0.1 even when A is not a commutative subalgebra of U. We prove that if U is a central separable n^2-dimensional k algebra and $\tilde{U}$ is *any* n^2-dimensional k-algebra then there is a 2-cocycle in $U \otimes U \otimes U$ making U isomorphic to $\tilde{U}$ (via 0.1). Moreover we show that if U is a central separable k algebra with simple subalgebra L which has centralizer A then there is a 2-cocycle in $A \otimes A \otimes A$ making U isomorphic to $\tilde{U}$ if and only if $\tilde{U}$ contains a copy of L and is isomorphic to U as an L-bimodule. If A is commutative and σ is a 2-cocycle in $A \otimes A \otimes A$ then σ is an Amitsur 2-cocycle if σ is invertible.

We define when two 2-cocycles in $A \otimes A \otimes A$ are cohomologous and show that this is equivalent to the associated algebras being isomorphic by an isomorphism which is the identity on L. This gives a bijective correspondence between a 2-cohomology set (not group) and equivalence classes of algebras.

1. Linear Algebra

Throughout this paper k is at least a commutative unitary ring (and sometimes a field). All k algebras are unitary. A subalgebra has the same unit

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