ON THE RELATIVE STRENGTH OF KARAMATA MATRICES

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1. Introduction and definitions

If $A = (a_{nk})$ is an infinite matrix and $S = \{S_k\}$ is a sequence then A is *applicable* to S if all of the series

$$y_n = \sum_{k=0}^{\infty} a_{nk} S_k, \qquad n = 0, 1, 2, \cdots$$

converge. If, in addition, $\{y_n\}$ is a convergent sequence, then A *sums* S to $\lim y_n$. Whenever A sums every convergent sequence S to $\lim S_n$ then A is *regular*. If A and B are two matrices and A sums every sequence that B sums then A is *stronger* than B.

Many useful matrices in the theory of summability are obtained from non-constant functions $f(z)$ that are analytic in a neighborhood of the origin by setting

$$[f(z)]^n = \sum_{k=0}^{\infty} f_{nk} z^k, n = 1, 2, \cdots, \qquad f_{00} = 1, \qquad f_{0k} = 0, k = 1, 2, \cdots.$$

The matrix (f_{nk}) is said to be generated by $f(z)$. For example $f(z) = 1 - r + rz$ generates the Euler E_r method [1] and $f(z) = (1 - r)/(1 - rz)$ generates a method studied by W. Meyer-König [3] and P. Vermes [6].

A natural generalization of these methods is the matrix generated by

$$f(z) = (\alpha + (1 - \alpha - \beta)z)/(1 - \beta z).$$

Such a method is called a Karamata matrix and will be denoted by $K[\alpha, \beta]$. B. Bajsanski [2] has studied these matrices and determined conditions that they be regular when α and β are real. He has also investigated the relative strength of different Karamata matrices.

The conditions for regularity of $K[\alpha, \beta]$ have been generalized to complex values of α and β [5], and Bajsanski's theorem about the relative strength of Karamata matrices is a corollary of a more powerful result in his paper. It is then reasonable to hope that more specialized techniques together with more information on regularity will yield other theorems about the relative strength of Karamata matrices. Results of this type will be found in Section 3. Section 2 is devoted to some preparatory theorems, and Section 4 to closing remarks.

2. Some preparatory theorems

The Weierstrass theorem on uniformly convergent series of analytic functions will be a primary tool, and when reference is made to Weierstrass' theorem it is this theorem which is being cited.

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