RESIDUAL PROPERTIES FOR CLOSED OPERATORS ON FRÉCHET SPACES

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1. Introduction

The single-valued extension property is a remarkable one for a very large class of linear operators on a locally convex space. Its first definition is due to Dunford and is related to spectral operators, which possess this property [5], [6]. But there exist simple examples of operators which do not have this property or have it only on a part of their spectrum. So it is very natural to consider a residual part of the spectrum of an operator and define this property "outside" this part [11]. Many properties of a spectral operator (and even for an operator in certain larger classes [7], [4]) can be obtained using only a natural assumption of decomposability of the space with respect to this operator [8]. To study simultaneously the class of unbounded operators with a suitable spectral behaviour and other classes (obtained for example, from direct sums between "good" operators and operators which do not even have the single valued extension property) it is again necessary to consider a residual part of their spectrum [11]. The purpose of our paper is to give some new results related to the single-valued extension property and supplementary assertion for the residually decomposable operators [11].

Our main result is a theorem of existence and uniqueness, for a large class of operators, of a minimal closed set outside which such an operator has a suitable spectral behaviour.

First we need some definition and additional properties. In the sequel $\mathfrak{X}$ will be a Fréchet space [3] (although many considerations are true in more general spaces), $B(\mathfrak{X})$ the set of all continuous linear operators on $\mathfrak{X}$, and $C(\mathfrak{X})$ the set of all closed linear operators on $\mathfrak{X}$. For the spectrum $\sigma(T)$ of an operator $T \in C(\mathfrak{X})$ we shall use the definition of Waelbroeck [12]. Thus a point $\lambda \in \mathbf{C}_\infty (= \mathbf{C} \cup \{\infty\})$ is in $\rho(T)$ if there exists a neighbourhood $V_\lambda \subseteq \mathbf{C}_\infty$ of λ such that $(\mu I - T)^{-1} \in B(\mathfrak{X})$ for any $\mu \in V_\lambda \cap \mathbf{C}$ and the set

$$\{(\mu I - T)^{-1} x;\ \mu \in V_\lambda \cup \mathbf{C}\}$$

is bounded in $\mathfrak{X}$ for any $x \in \mathfrak{X}$. We shall also use the well-known notations: $R(\lambda, T) = (\lambda I - T)^{-1}$, $\mathfrak{D}_T$ for the domain of the operator T and $\sigma(T) = \complement\rho(T)$ (all operations with sets are considered in $\mathbf{C}_\infty$).

Let $T \in C(\mathfrak{X})$ and $x \in \mathfrak{X}$ be fixed. We shall say that $\lambda \in \delta_T(x)$ if in a neighbourhood V_λ of λ there exists an analytic function $f_x : V_\lambda \to \mathfrak{D}_T$ (not necessarily unique) such that $(\mu I - T) f_x(\mu) = x$ for $\mu \in V_\lambda \cap \mathbf{C}$. Such an analytic func-

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