THE AUTOMORPHISMS OF THE UNITARY GROUPS AND THEIR CONGRUENCE SUBGROUPS

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The automorphism theory of the unitary group and the projective unitary group has been investigated only over fields. In particular, no automorphism theory has been available for the groups of unitary matrices whose entries are taken from an integral domain. In this paper, we consider subgroups G of the projective unitary group $PU_n(V)$ such that for each isotropic line L of V, G contains at least one non-trivial projective transvection with proper line L.

We require the underlying hermitian form to have Witt index at least 3. We then give group-theoretical properties of the projective unitary transvections in G which suffice to distinguish the projective transvections from the other projective unitary transformations. From this it follows that an automorphism Λ of G must preserve all projective unitary transvections. This yields a bijection $L \to L'$ of the isotropic lines of the underlying vector space V; we extend this to a bijection of all the totally isotropic subspaces of V and then apply a theorem of Chow and Dieudonné [3, p. 82] to conclude the bijection of totally isotropic subspaces is induced identically by a unitary semi-similitude g of V. It is then easy to show the automorphism Λ is given by transformation by the projective semi-similitude $\bar{g}$ corresponding to g. Our results hold for Witt index at least 3 and characteristic not 2.

Having obtained the automorphisms of such projective unitary groups G we determine as a corollary all the automorphisms of any subgroup S of $U_n(V)$ that contains at least one non-trivial transvection on each isotropic line of V. In the final section we apply these results to the unitary groups U_n, U_n^+, T_n defined over integral domains and to their congruence subgroups. We show each such unitary congruence group contains a non-trivial transvection on every isotropic line. Applying our previous results we thus obtain an automorphism theory for the unitary congruence groups.

The techniques used in this paper are modifications of the original method of residual spaces introduced by O'Meara in [6] for the congruence subgroups of the special linear and general linear groups.

1. Preliminaries

Let V be an n-dimensional vector space over the field F, and let $\chi(F)$ denote the characteristic. Consider a hermitian form (x, y) on V, i.e., a map

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