ON THE IMAGE OF $S^p \times S^q$ UNDER MAPPINGS OF DEGREE ONE

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0. Introduction

This paper computes the homotopy type of those closed, connected, orientable, topological $(p + q)$-manifolds which admit a degree 1 mapping from $S^p \times S^q$ for $p, q \geq 1$. The principal result is

THEOREM 2. *Let M be a closed, connected, orientable, topological $(p + q)$-manifold. If M admits a degree 1 mapping $f : S^p \times S^q \to M$, then either M has the homotopy type of S^{p+q}, or f is a homotopy equivalence.*

This theorem is analogous to the following results, which appear in [2, 2.6 and 2.7, pp. 216–217].

PROPOSITION. *Let M be a closed, orientable, topological or piecewise linear n-manifold, $n \geq 5$. If there is a degree 1 map $S^n \to M$, then M is isomorphic to S^n.*

THEOREM. *Let M be an unbounded, orientable, differentiable or piecewise linear n-manifold, $n \geq 5$. If there is a proper degree 1 map $R^n \to M$, then M is isomorphic to R^n.*

1. The degree of a map

If M and N are connected, orientable n-manifolds, then

$$H_c^n(M, \partial M) = H_c^n(N, \partial N) = Z,$$

where Z denotes the infinite cyclic group. (H_c^* denotes the integral singular cohomology based on cochains with compact support.) If μ_M and μ_N are the preferred free generators of the groups above, then the degree of a proper map

$$f : (M, \partial M) \to (N, \partial N)$$

is the integer k satisfying

$$f^*(\mu_N) = k\mu_M .$$

The proof of Theorem 2 requires repeated use of the following fundamental lemma, proved in [2, 2.9 and 2.11, pp. 216–217].

LEMMA 1. *If $f : (M, \partial M) \to (N, \partial N)$ is a proper mapping of degree* 1

Received October 30, 1970.

[1] This work forms part of the author's doctoral thesis written under the supervision of Professor K. W. Kwun at Michigan State University. It was supported in part by a National Science Foundation grant.