# AN ARC THEOREM FOR PLANE CONTINUA

BY

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If $H$ is a bounded aposyndetic plane continuum which does not separate the plane, then $H$ is locally connected. This follows from a result of Jones' [3, Th. 10] that if $p$ is a point of a bounded plane continuum $H$ and $H$ is aposyndetic at $p$, then the union of $H$ and all but finitely many of its complementary domains is connected im kleinen at $p$.[1] As a corollary of these results, each bounded aposyndetic nonseparating plane continuum is arc-wise connected. Closely related to the notion of an aposyndetic continuum is that of a semi-aposyndetic continuum, studied in [2]. A continuum $M$ is *semi-aposyndetic* if for each pair of distinct points $x$ and $y$ of $M$, there exists a subcontinuum $F$ of $M$ such that the sets $M - F$ and the interior of $F$ relative to $M$ each contain a point of $\{x, y\}$. Note that a bounded semi-aposyndetic nonseparating plane continuum may fail to be locally connected. In this paper it is proved that every bounded semi-aposyndetic nonseparating plane continuum is arc-wise connected.

Throughout this paper $S$ is the plane and $d$ is the Euclidean metric for $S$.

DEFINITION. Let $E$ be an arc-segment (open arc) in $S$ with endpoints $a$ and $b$, $D$ be a disk in a continuum $M$ in $S$, and $\varepsilon$ be a positive real number. The arc-segment $E$ is said to be *$\varepsilon$-spanned* by $D$ in $M$ if $\{a, b\}$ is a subset of $D$ and for each point $x$ in a bounded complementary domain of $D \cup E$, either $d(x, E) < \varepsilon$ or $x$ belongs to $M$.

LEMMA 1. *If an arc-segment $E$ in $S$ of diameter less than $\varepsilon$ with endpoints $a$ and $b$ is $\varepsilon$-spanned by a disk $D$ in $M$ (a subcontinuum of $S$), then there exists an arc-segment $M(E)$ in $M$ with endpoints $a$ and $b$ such that for each point $x$ of $M(E)$, $d(x, E) \leqq 2\varepsilon$.*

*Proof.* Let $w$ be a point of the unbounded complementary domain of $D \cup E$. Let $B$ denote an arc in $D$ with endpoints $a$ and $b$. For each positive real number $r$, let $C(r)$ denote the set consisting of all points $x$ of $S$ such that $d(x, \text{Cl } E) < r$ (Cl $E$ is the closure of $E$). For each positive real number $r$, Cl $C(r)$ is a bounded locally connected continuum in $S$ which does not contain a separating point. By a simple argument, one can show that if $r \geq \varepsilon$, Cl $C(r)$ does not separate $S$. Hence for each real number $r \geq \varepsilon$, Cl $C(r)$ is a disk [5, Th. 4, p. 512]. Since $B$ is locally connected, the set $Q$ consisting of all components of $B -$ Cl $E$ which meet $Bd\ C(\varepsilon)$ (the boundary

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[1] A continuum $H$ is *aposyndetic* at a point $p$ of $H$ if for each point $q$ of $H - \{p\}$, there exist a continuum $L$ and an open set $G$ in $H$ such that $p \in G \subset L \subset H - \{q\}$. A continuum is said to be *aposyndetic* if it is aposyndetic at each of its points (Jones).