DIRICHLET SPACES AND RANDOM TIME CHANGE

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Introduction

We apply the machinery of Dirichlet spaces to the study of processes obtained from a given symmetric Markov process via a random time change. The basic results are collected and presented in their final form in Section 8. These results are applicable in particular to the "processes on the boundary" of K. Sato and T. Ueno in [25] and so yield information about boundary conditions for symmetric Markov processes.

Necessary preliminary machinery is set up in Sections 1 through 4. Some potential theory for regular Dirichlet spaces as developed for special cases by H. Cartan [3 and 4] and in general by A. Beurling and J. Deny [1] and by M. Fukushima [13] is collected in Section 1 and then applied in Section 2 to construct strong Markov processes. Our construction differs from that of Fukushima [13] in that we do not make use of "strongly regular Dirichlet spaces" and the associated "Ray resolvents." The increasing processes for implementing random time changes are constructed in Section 3. Also in that section we introduce an appropriate notion of balayage more or less in the spirit of [13]. A useful form of invariance under time reversal is established in Section 4.

The real work is done in Sections 5 and 6. The time changed process is analyzed in Section 5. The main result is that the corresponding Dirichlet space is, in an appropriate sense, contained in a "universal Dirichlet space" which itself depends only on a complementary "killed" process and topological notions. A converse result is established in Section 6 so that these two sections together effectively classify certain symmetric extensions of the "killed" process.

Probabilistic interpretations for the original Dirichlet norm and also for the "universal Dirichlet norm" are established in Section 7. The results here are incomplete but promise to be useful for applications.

Our results are illustrated with Brownian motion in Section 9. In particular we show that a result in Section 7 gives a general form of Green's identity, similar to one established by Doob [5] for the Martin closure using different techniques.

We plan to treat processes of purely "jump type" in a separate publication.

For related work by other authors we refer to [8], [14], [18], [25] and [26].

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