THE TODD CHARACTER AND THE INTEGRALITY THEOREM FOR THE CHERN CHARACTER

BY

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In [5] the authors introduced a homomorphism called the Todd character, relating complex bordism homology theory to rational homology. Specifically, the Todd character is a family of homomorphisms

$$th : MU_n(X) \rightarrow \oplus_{k=0}^{[n/2]} H_{n-2k}(X; Q),$$

the component morphisms being denoted by

$$th_{n-2k} : MU_n(X) \rightarrow H_{n-2k}(X; Q).$$

By analogy with the Chern character we may seek an integrality theorem for the Todd character. This turns out to be a comparatively easy task given a few elementary facts concerning the classical Todd polynomials. The precise result that we will establish is:

INTEGRALITY THEOREM FOR THE TODD CHARACTER. *Let X be a cw-complex and $\alpha \in MU_{2n}(X)$. Then*

$$\mu_{n-k} th_{2k}(\alpha) \in H_{2k}(X; Z)/\text{torsion} \subset H_{2k}(X; Q)$$

where μ_t is the integer defined by

$$\mu_t = \prod_{\text{primes}} p^{[t/p-1]}.$$

(A similar result holds for odd dimensional classes.)

In [5] and [6] several applications of the Todd character were made and it was often remarked that the Todd character and the Chern character are closely related. We will display one such relation in the present note by deriving the integrality theorem of J. F. Adams [1] for the Chern character from the corresponding result for the Todd character and elementary properties of the complex bordism homology and cohomology theories. Our treatment of the integrality theorem for the Chern character seems closely related to the special case treated by E. Dyer in [7].

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1. Definition and elementary properties of the Todd character

Let us begin by recalling the definition of the Todd character

$$th : MU_{**}(\quad) \rightarrow H_{**}(\quad ; Q)$$

as given in [5].

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