FINITE GROUPS WHOSE SYLOW 2-SUBGROUPS ARE THE DIRECT PRODUCT OF A DIHEDRAL AND A SEMI-DIHEDRAL GROUP

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1. Introduction

The purpose of this paper is to classify all finite fusion-simple groups which have a Sylow 2-subgroup that is the direct product of a dihedral group with a semi-dihedral group. (We say that a group G is *fusion-simple* if $O^2(G) = G$ and $Z^*(G) = 1$. A *semi-dihedral group* is also known as a quasi-dihedral group.) Our main result is as follows:

THEOREM. *Let G be a finite fusion-simple group with a Sylow 2-subgroup that is the direct product of a dihedral group and a semi-dihedral group. Then G has a normal subgroup of odd index of the form $F_1 \times F_2$ where*

$$F_1 \cong A_7, \quad PSL(2, q_1),\ q_1 \text{ odd},\ q_1 \geq 5, \quad \text{or} \quad Z_2 \times Z_2$$

and

$$F_2 \cong M_{11}, \quad PSL(3, q_2),\ q_2 \equiv -1 (\text{mod } 4),\ \text{or } PSU(3, q_2),\ q_2 \equiv 1 (\text{mod } 4).$$

The essential ideas used in proof are to be found in [6]. In particular, we assume that a group G is a minimal counter-example to our theorem. We then show that G has an involution fusion pattern compatible with the conclusion of the theorem. Next, we select an arbitrary elementary abelian subgroup A of order 16 in G. Then for suitable four-groups X and Y contained in A such that $A = X \times Y$, we establish the following assertion:

If for $a \in A^{\#}$, one sets

$$\theta(C_G(a)) = \langle C_G(a) \cap O(C_G(x)) \cap O(C_G(y)) \mid x \in X^{\#},\ y \in Y^{\#} \rangle,$$

then θ is an *A-signalizer functor* on G in the sense of Goldschmidt [4].

If θ is nontrivial, we conclude that $W_A = \langle \theta(C_G(a)) \mid a \in A^{\#} \rangle$ is a group of odd order and this allows us to show that $N_G(W_A)$ is a strongly imbedded subgroup of G. It then easily follows that θ is trivial and from this we prove that G satisfies the conclusions of our theorem. This contradiction then proves our theorem.

We use the following definitions which are slight restrictions of some definitions in [2]:

(i) A finite group G is said to be an *SD-group* if a Sylow 2-subgroup of G is a semi-dihedral group and G contains one conjugacy class of involutions and one conjugacy class of elements of order 4.

(ii) A finite group G is said to be a *Q-group* if a Sylow 2-subgroup of G

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