NOTE ON A CRITERION OF SCHEERER

BY

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1. Introduction

In [4] Scheerer considered principal G-bundles over S^n

$$G \to E_\alpha \xrightarrow{f_\alpha} S^n, \tag{1.1}$$

classified by $\alpha \in \pi_{n-1}(G)$, and proved the following theorem.

THEOREM 1.1. *Suppose the diagram*

$$\begin{array}{ccc} S^{n-1} \times G & \xrightarrow{\mu(\alpha \times 1)} & G \\ \downarrow {\scriptstyle 1 \times k} & & \downarrow {\scriptstyle k} \\ S^{n-1} \times G & \xrightarrow{\mu(k\alpha \times 1)} & G \end{array} \tag{1.2}$$

is (homotopy) commutative, where $k : G \to G$ is the k^{th} power map and $\mu : G \times G \to G$ is the multiplication. Then

$$k\alpha_0 \circ f_\alpha = 0 \tag{1.3}$$

where $\alpha_0 : S^n \to B_G$ is adjoint to α.

Now consider the pull-back diagram

$$\begin{array}{ccc} \bar{E} & \longrightarrow & E_{k\alpha} \\ \downarrow & & \downarrow {\scriptstyle f_{k\alpha}} \\ E_\alpha & \xrightarrow{f_\alpha} & S_n \end{array} \tag{1.4}$$

Then, of course, (1.3) guarantees that

$$\bar{E} = E_\alpha \times G, \tag{1.5}$$

so that Theorem 1.1 is highly relevant to the study of non-cancellation phenomena[1] in [1], [2], [3], [4]. Indeed in [2] it is shown that the hypothesis of Theorem 1.1 above is equivalent, in the case $G = S^3$, to the key condition

$$\tfrac{1}{2}\, k(k - 1)\omega \circ \Sigma^3\alpha = 0 \in \pi_{n+2}(S^3) \tag{1.6}$$

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[1] A different, but related, approach to non-cancellation phenomena is due to A. Sieradski.