# SOME GENERALIZATIONS, TO CERTAIN LOCALLY FINITE GROUPS, OF THEOREMS DUE TO CHAMBERS AND ROSE

BY

C. J. GRADDON

## 1. Introduction

In this paper we examine further the properties of the class $\mathfrak{U}$ introduced in [4] and studied again in [9] and [10]. We refer the reader to these papers for our notation and terminology. The class $\mathfrak{U}$ resembles in many ways the class of finite soluble groups. Indeed a result for finite soluble groups which makes sense in the wider context usually holds for $\mathfrak{U}$-groups. This is certainly the case with Gaschutz' theory of formations which was carried over to arbitrary $QS$-closed subclasses $\mathfrak{K}$ of $\mathfrak{U}$ by Gardiner, Hartley and Tomkinson [4]. We have also extended [7] our earlier work [5], [6] on $\mathfrak{F}$-reducers and $\mathfrak{F}$-subnormalizers to such classes $\mathfrak{K}$. In addition the results of Alperin [1] on system normalizers, Carter subgroups and the relation between them in finite soluble groups, were extended to $\mathfrak{U}$-groups in [9]. It is our aim to show here that many of the results of Chambers [3] and Rose [15] hold in appropriate subclasses of $\mathfrak{U}$. Generalizing Chambers [3] we show, for example, that if $\mathfrak{K}$ is a $QS$-closed subclass of $\mathfrak{U}$ and $\mathfrak{F}$ a saturated $\mathfrak{K}$-formation then the $\mathfrak{F}$-normalizers are pronormal in $\mathfrak{K}_A$-groups (i.e. $\mathfrak{K}$-groups with abelian Sylow $p$-subgroups for each prime $p$). This yields a partial extension of Alperin's [1, Theorem 1] for $\mathfrak{F}$-normalizers and $\mathfrak{F}$-projectors of $\mathfrak{K}_A$-groups. We shall also show that the $\mathfrak{F}$-normalizers of $\mathfrak{K}_A$-groups are characterized as those subgroups which cover the $\mathfrak{F}$-central and avoid the $\mathfrak{F}$-eccentric chief factors. We shall extend Chambers' work in Section 2 and Rose's in Section 3. In the third and final section we shall consider the class $\mathfrak{D}$ of $\mathfrak{U}$-groups with pronormal basis normalizers. For example, we prove that $\mathfrak{D}$ is a $\mathfrak{U}$-formation (in the sense of [4, §1]) and derive many of its properties from our work [7] on reducers in $\mathfrak{U}$-groups.

## 2. $\mathfrak{U}$-groups with abelian Sylow subgroups

If $\mathfrak{X}$ is a subclass of $\mathfrak{U}$ we denote by $\mathfrak{X}_A$ the class of $\mathfrak{X}$-groups with abelian Sylow $p$-groups for each prime $p$. In this section we study the class $\mathfrak{U}_A$ showing in particular that most of Chambers' results on finite soluble $A$-groups can be extended to the class $\mathfrak{U}_A$ or appropriate subclasses of it.

It is clear that if $\mathfrak{X}$ is a $QS$-closed subclass of $\mathfrak{U}$ then so is $\mathfrak{X}_A$ (cf. [4, 2.1]).

LEMMA 2.1. *Every $\mathfrak{U}_A$-group is soluble.*

*Proof.* If $G \in \mathfrak{U}_A$ then $G$ has a finite normal series with locally nilpotent factors. Since $\mathfrak{U}_A$ is $QS$-closed and every locally nilpotent $\mathfrak{U}_A$-group is abelian, each of these factors is abelian. Hence $G$ is soluble as claimed.

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