BORDISM J-HOMOMORPHISMS

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1. Introduction

In these notes we introduce various generalizations of the bordism J-homomorphisms of [3] and [6]. We compute ImJ in a few of the easier cases, and discuss the geometrical implications of our results. For all notation, see [7].

In §2, we set up a diagram of the form

$$\begin{array}{ccc} \pi_*(G_i/G_j) & & \\ \downarrow & \searrow^{J} & \\ & & (MG_j)_* \\ & \nearrow_{J} & \\ (MG_k)_*(G_i/G_{j+}) & & \end{array}$$

for each triple of stable subgroups $G_k \leq G_j < G_i$ of Top. The case $G_i = F$, homotopy equivalences of the sphere, may often be incorporated. Both $\pi_*(G_i/G_j)$ and $(MG_k)_*(G_i/G_{j+})$ admit geometrical interpretations, and we also discuss the J-homomorphisms from this angle. A host of examples is given in §3.

We set up in §4 a homological formula. This eases our calculations with the MU_* J-homomorphisms, the cases we shall mainly deal with. Rationally, the sums are easy, so we do them in §5. Their simplicity notwithstanding, they do in many cases give good insight into the structure of ImJ.

Thus fortified, in §§6 and 7 we compute completely the images of $J:\pi_*(SO/U) \to MU_*$ and $J:MU_*(SO/U) \to MU_*$. In neither case is the image a direct summand. In §8, we explain our results in terms of unitary structures on manifolds.

The development of the ideas herein was much influenced by many enjoyable discussions with Bob Switzer and Reg Wood. Idar Hansen was also very helpful.

2. Constructions

Let us first consider a triple of stable subgroups $G_k \leq G_j < G_i$ of the orthogonal group O. We shall later, with due care, extend many of our constructions to include the cases PL, Top and F.

Choose integers ε and ε_i such that $G_i(\varepsilon_i n)$ and $G_j(n)$ act on $\mathbf{R}^{\varepsilon n}$ (e.g. if $G_i = U$ and $G_j = Sp$ then $\varepsilon = 4$ and $\epsilon_i = 2$). Then as explained in [6],

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