MORE ON HIGH-ORDER NON-LOCAL UNIFORM ALGEBRAS

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1. Introduction

Consider the process of taking the uniform closure of the functions belonging locally to a uniform algebra. After how many iterations of this process does the resulting (transfinite) sequence of uniform algebras stabilize? The possibilities turn out to be the finite and countable ordinal numbers, for each of which an example with five generators is constructed, and the first uncountable ordinal number, for which each example must be non-separable. A similar result, with four generators instead of five, is true for the process of taking the uniform closure of the functions locally approximable by a uniform algebra. These theorems improve previous results of the author [6], both by extending (and determining) the number of iterations possible, and, for the second process, by introducing control over the number of generators of the algebras involved.

In §2, certain notions are recalled and the main results of the paper are stated. The proofs are given in §3 and §4. For many details, the reader will be referred to [6]. Finally, §5 is devoted to indicating a number of open problems concerning non-local algebras.

2. The main results

If (A, X) is a uniform algebra with spectrum X, we denote by $L(A)$ and $H(A)$ the respective closures of the functions locally belonging to A and the A-holomorphic functions (that is, the functions locally (uniformly) approximable by A). A is said to be local or non-local according as $L(A) = A$ or $L(A) \neq A$, and holomorphically closed or non-holomorphically closed according as $H(A) = A$ or $H(A) \neq A$. We can inductively define $L^\sigma(A)$ and $H^\sigma(A)$ for all ordinal numbers σ by the rules $L^0(A) = H^0(A) = A, L^{\sigma+1}(A) = L(L^\sigma(A))$ and $H^{\sigma+1}(A) = H(H^\sigma(A))$, and, if σ is a limit ordinal, $L^\sigma(A)$ is the uniform closure of

$$\bigcup\{L^{\sigma'}(A) : 0 \leq \sigma' < \sigma\}$$

and $H^\sigma(A)$ is the uniform closure of

$$\bigcup\{H^{\sigma'}(A) : 0 \leq \sigma' < \sigma\}.$$

Recall that for all σ, $(L^\sigma(A), X)$ and $(H^\sigma(A), X)$ are uniform algebras with spectrum X (see [6], §2). Let σ^* denote the first uncountable ordinal number.

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