APPLICATIONS OF NULL-HOMOTOPIC SURGERY

BY

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In the first part of this paper we study a class of manifolds obtained by attaching $S^{n-1} \times D^n$ to $D^n \times S^{n-1}$ with a diffeomorphism of the boundary. The class we study is those manifolds with the attaching diffeomorphism of the following form. Let $f : S^{n-1} \to SO(n)$ and $g : S^{n-1} \to SO(n)$; then

$$h : S^{n-1} \times S^{n-1} \to S^{n-1} \times S^{n-1}$$

is defined by $h(x, y) = ([f(g(x)y)]^{-1}x, g(x)y)$. We denote the resulting manifold by $M(f, g)$. This class is motivated by looking at the boundary of the manifold obtained when two copies of the tangent disk bundle of S^n are plumbed together. We then study the manifolds which result when surgery is done on the S^{n-1} factor of the right hand side, the natural framing being twisted by a function $t : S^{n-1} \to SO(n)$. Using these results and a generalization of a theorem we proved in [3], we obtain an elementary proof of a result which generalizes the following theorem due to E. H. Brown, Jr. and B. Steer [1].

THEOREM. *Suppose n is odd, $n \neq 1, 3, 7$, V_n is the Stiefel manifold of unit tangent vectors to S^n, and Σ^{2n-1} is the sphere obtained by plumbing two copies of the tangent disk bundle of S^n. Then V_n and $V_n \# \Sigma^{2n-1}$ are diffeomorphic.*

In the last section of the paper we show how the ideas and techniques of the preceding sections can be used to give an elementary proof of some of the results of Tamura [5].

The proofs in this paper are more elementary than the ones given in the above cited papers in that we are able to explicitly define diffeomorphisms between manifolds whose existence in the papers was obtained by the h-cobordism theorem.

1. The homology of $M(f, g)$

In this section we will analyze the homology of the manifold obtained from $D^n \times S^{n-1} \cup S^{n-1} \times D^n$ where the identification on the boundary is given by

$$(x, y) \to ([f(g(x)y)]^{-1}x, g(x)y).$$

Here as in the introduction $f, g : S^{n-1} \to SO(n)$. We denote the resulting manifold by $M(f, g)$. Since we have the fibration

$$SO(n-1) \to SO(n) \xrightarrow{\pi} S^{n-1}.$$

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