ON FINITE GROUPS OF COMPONENT TYPE

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In recent years great progress has been made toward the classification of finite simple groups in terms of local subgroups and in particular the centralizers of involutions. If this program is to be completed one must show that an arbitrary simple group G possesses an involution t for which $C_G(t)$ is isomorphic to a centralizer in a known simple group.

This paper concerns itself with that problem for simple groups of *component type*; that is groups G such that $E(C(t)/O(C(t))) \neq 1$ for some involution t in G. These include most of the Chevalley groups of odd characteristic, most of the alternating groups, and many of the sporadic simple groups. D. Gorenstein has conjectured that in a group of component type, the centralizer of some involution is usually in a "standard form." A proof is supplied here of a portion of that conjecture.

To be more precise, define a subgroup K of a finite group G to be *tightly embedded* in G if K has even order while $K \cap K^g$ has odd order for each $g \in G\text{-}N(K)$. Define a quasisimple subgroup A of G to be *standard* in G if $[A, A^g] \neq 1$ for each $g \in G$, $K = C_G(A)$ is tightly embedded in G, and $N(A) = N(K)$.

Let G be a finite simple group of component type in which $O_{2',E}(C(t)) = O(C(t))E(C(t))$ for each involution t in G. Let A be a "large component." Then it is shown, modulo a certain special case where A has 2-rank 1, that A is standard in G in the sense defined above.

Other theorems establish properties of tightly embedded subgroups. They show that, under the hypothesis of the last paragraph, the centralizer of each involution centralizing A contains at most one component distinct from A, and that component must have 2-rank 1 if it exists. Further, it can be shown that the 2-rank of the centralizer of A is bounded by a function of A, which seems to be 1 or 2 if A is not of even characteristic.

Proofs of the various theorems utilize properties of the Generalized Fitting Subgroup $F^*(G)$ of a group G, developed by Gorenstein and Walter. These properties appear in Section 2. Also important to the proof is the classification of groups with dihedral Sylow 2-groups, Alperin's fusion theorem, the recent result on 2-fusion due to Goldschmidt, and Theorem 3.3 in Section 3, which extends Bender's classification of groups with a strongly embedded subgroup.

Statements of the major theorems appear in Section 1, along with a brief explanation of notation.

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