PRODUCTS OF WITT GROUPS

BY

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Introduction

We study products of Witt groups and answer the question of whether a direct factor of a product of Witt groups is also a product of Witt groups affirmatively.

The proof proceeds after an examination of the extendability of endomorphisms of products of Witt groups, and uses the cohomological study of algebraic group extensions given by Serre in *Groupes Algébriques et Corps de Classes.*

The category is that of affine algebraic groups.

0. Facts from Serre (Chapter 7)

Let k be an algebraically closed field.

(1) For commutative connected algebraic groups A and B, the group $\mathrm{Ext}\,(A, B)$ of classes of (commutative) extensions of A by B is the group $H^2_{\mathrm{reg}}\,(A, B)_S$ of classes of symmetric regular factor sets $f\colon A \times A \to B$ [Serre, Proposition 7, Number 6]. We will abbreviate this group to $H^2(A, B)$.

Let k have characteristic $p \neq 0$.

(2) For W_n the n-dimensional Witt group ($W_1 = G_a$, the additive group) and A_n the ring of endomorphisms of W_n, $H^2(W_n, G_a)$ is a right A_n-module and a left A_1-module under $\{f\}\cdot x = \{f \circ (x, x)\}$ and $y \cdot \{f\} = \{y \circ f\}$, for $\{f\}$ the element of $H^2(W_n, G_a)$ represented by the factor set f, $x \in A_n$ and $y \in A_1$.

(3) Let $F_n\colon W_n \times W_n \to G_a$ be a factor set with $F_n(0, 0) = 0$ which gives the extension

$$G_a \xrightarrow{V_n} W_{n+1} \xrightarrow{R} W_n,$$

$V^n(a) = (0, \dots, 0, a)$ and $R(a_0, \dots, a_n) = (a_0, \dots, a_{n-1})$. $H^2(W_n, G_a)$ is a free A_1-module on base $\{F_n\}$ [Serre, Lemma 4, No. 9].

(4) $H^2(\prod_j W_{n_j+1}, \prod_i {}_iG_a) = \prod_{i,j} H^2(W_{n_j+1}, {}_iG_a)$ [Serre, (10), No. 1].

Here each ${}_iG_a$ is the additive group. The connection is given by

$$\{F\} \mapsto (\{F_{ij}\})_{ij},$$

where

$$F_{ij}\colon W_{n_j+1} \times W_{n_j+1} \xrightarrow{\text{inc}} \prod_k W_{n_k+1} \times \prod_k W_{n_k+1} \xrightarrow{F} \prod_k {}_kG_a \xrightarrow{\text{proj}} {}_iG_a$$

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