GEOMETRY OF INTEGRAL SUBMANIFOLDS OF A CONTACT DISTRIBUTION

BY

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1. A differentiable $(2n + 1)$-dimensional manifold M is said to be a *contact manifold* if it carries a 1-form η such that $\eta \wedge (d\eta)^n \neq 0$. This condition, roughly speaking, means that the $2n$-dimensional "(tangent) subbundle" D defined by $\eta = 0$ is as far from being integrable as possible. In particular, the maximum dimension of an integral submanifold of D is n [3]. However, not much seems to be known about the immersion of such submanifolds into the ambient space, especially from Riemannian point of view. Thus we consider in this paper a normal contact metric (Sasakian) manifold, especially one with constant ϕ-sectional curvature, and study the immersion of its n-dimensional integral submanifolds.

The main result of this paper (Theorem 4.2) is that a compact minimal integral submanifold of a Sasakian space form M is totally geodesic if the square of the length of the second fundamental form is bounded by

$$\frac{n\{n(\tilde{c} + 3) + \tilde{c} - 1\}}{4(2n - 1)}$$

where $\tilde{c}$ is the ϕ-sectional curvature of M. In addition to giving other properties of integral submanifolds, we give examples in Section 5 of totally geodesic and minimal nontotally geodesic integral submanifolds.

2. Let M be a contact manifold with contact form η. It is well known that a contact manifold carries an *associated almost contact metric structure* (ϕ, ξ, η, G) where ϕ is a tensor field of type (1, 1), ξ a vector field, and G a Riemannian metric satisfying

$$\phi^2 = -I + \xi \otimes \eta, \quad \eta(\xi) = 1, \quad G(\phi X, \phi Y) = G(X, Y) - \eta(X)\eta(Y) \tag{2.1}$$

and

$$\Phi(X, Y) = G(X, \phi Y) = d\eta(X, Y). \tag{2.2}$$

The existence of tensors ϕ, ξ, η, G on a differentiable manifold M satisfying equations (2.1) is equivalent to a reduction of the structural group of the tangent bundle to $U(n) \times 1$ [2].

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